



Turnaj mladých fyzikov 2011: poznámky k niekoľkým úlohám *

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* prepáčte, ale všetky ostatné stránky budú v angličtine :-)

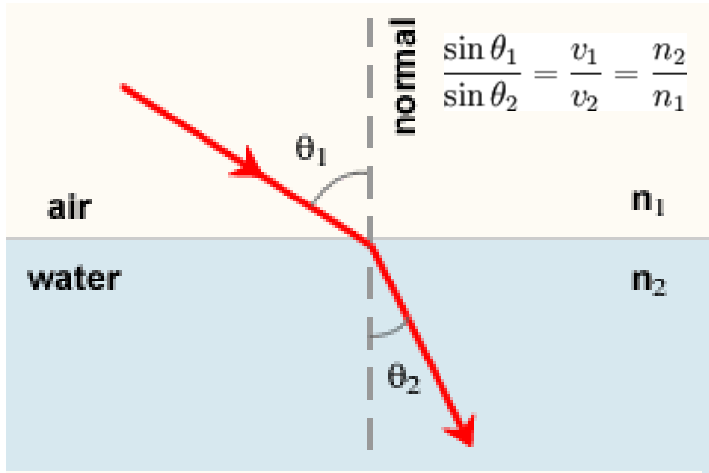
[Olli Niemitalo 2009]



Problem No. 11 “Fingerprints”

Fill a glass with a liquid and hold it in your hands. If you look from above at the inner walls of the glass, you will notice that the only thing visible through the walls is a very bright and clear image of patterns on your fingertips. Study and explain this phenomenon.

No fingers: nothing visible... why?



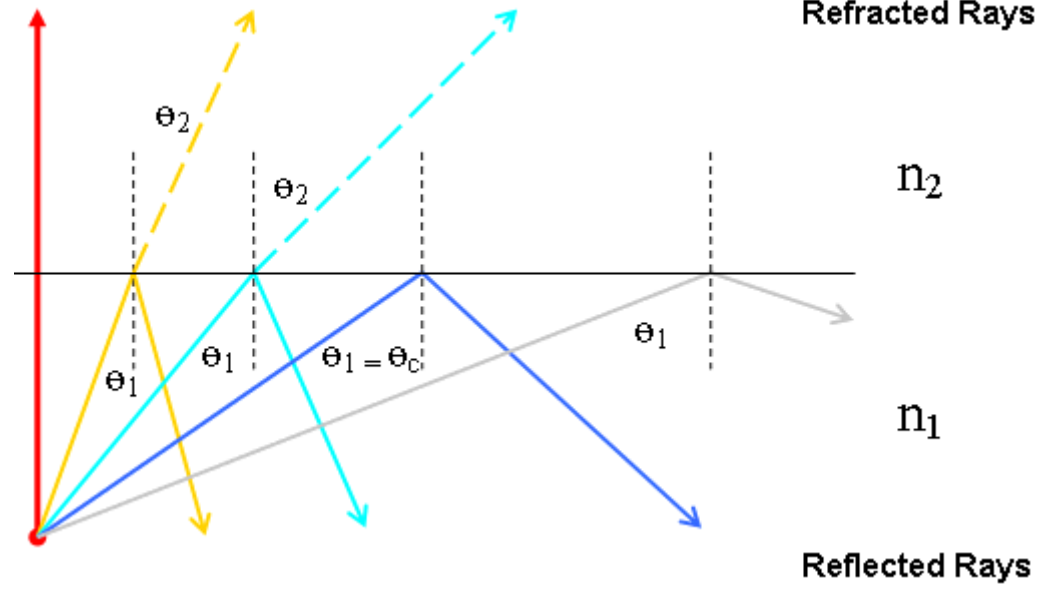
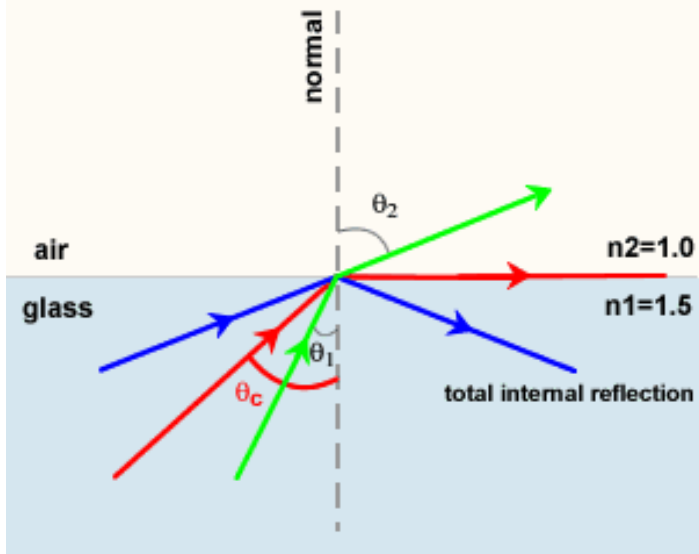
$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2} \sin \theta_1\right)$$

$$\theta_{\text{crit}} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$



Willebrord Snell
(1580-1626)

Refracted Rays





Total internal reflection

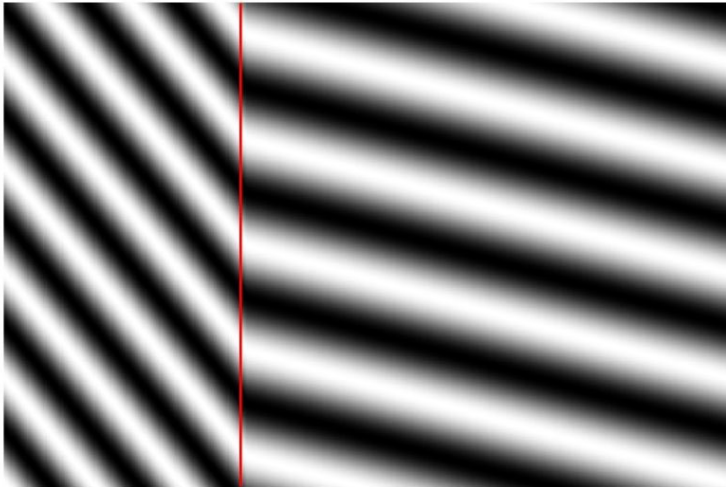


Underwater (total internal) reflection



Multiple total internal reflection

But little radiation still passes through...



- A weak, exponentially decaying wave exists behind the interface at the distances comparable to a single wavelength

■ → Evanescent wave

- Intensity vs distance from interface:

$$I(z) = I_0 \exp(-z/d).$$

$$d = \frac{\lambda}{4\pi} (n_2^2 \sin^2 \theta - n_1^2)^{-1/2}.$$



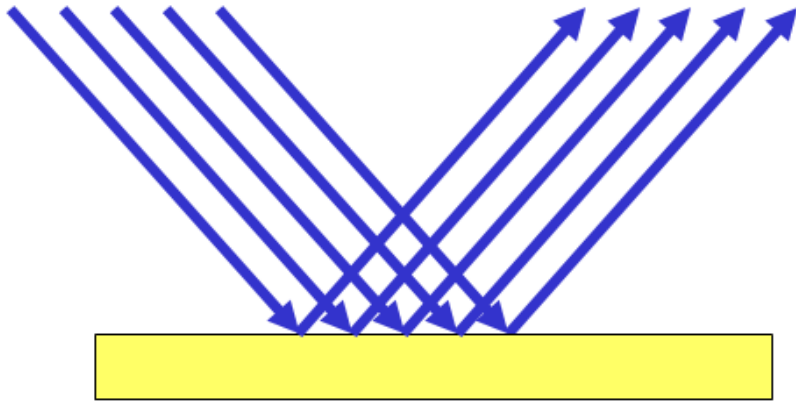
Your eyes are not a light source, and your fingers are not your eyes :-)

- The optical information is transmitted **from**, **not into** the “evanescent wave zone”
- → Fingers scatter light! :-) (what light?)
- How to correctly describe the system?

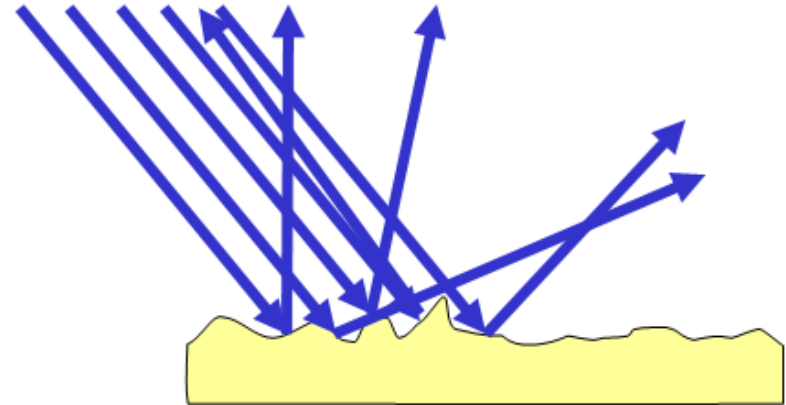
What is the approximate **gap** between the skin and the glass for the raised epidermal ridges, and for the grooves between them?

Reflection vs scattering

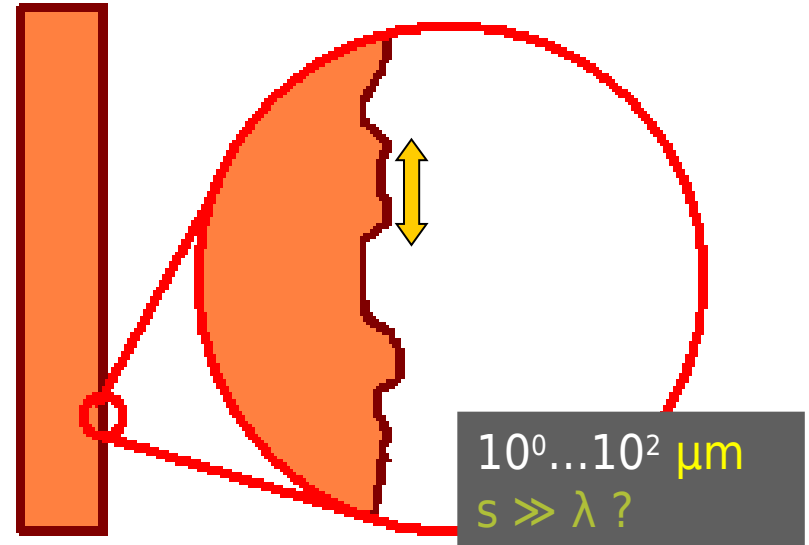
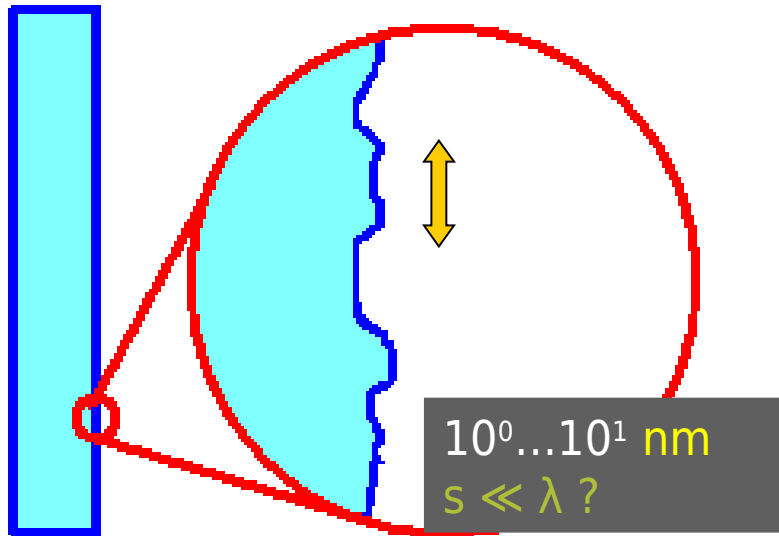
Specular Reflection



Diffuse Reflection



Roughness of glass and fingers...



- Noteworthy aspects:
 - ❑ rough glass looks turbid
 - ❑ rough surfaces never work as mirrors
 - ❑ rough surfaces scatter light



Contact area...

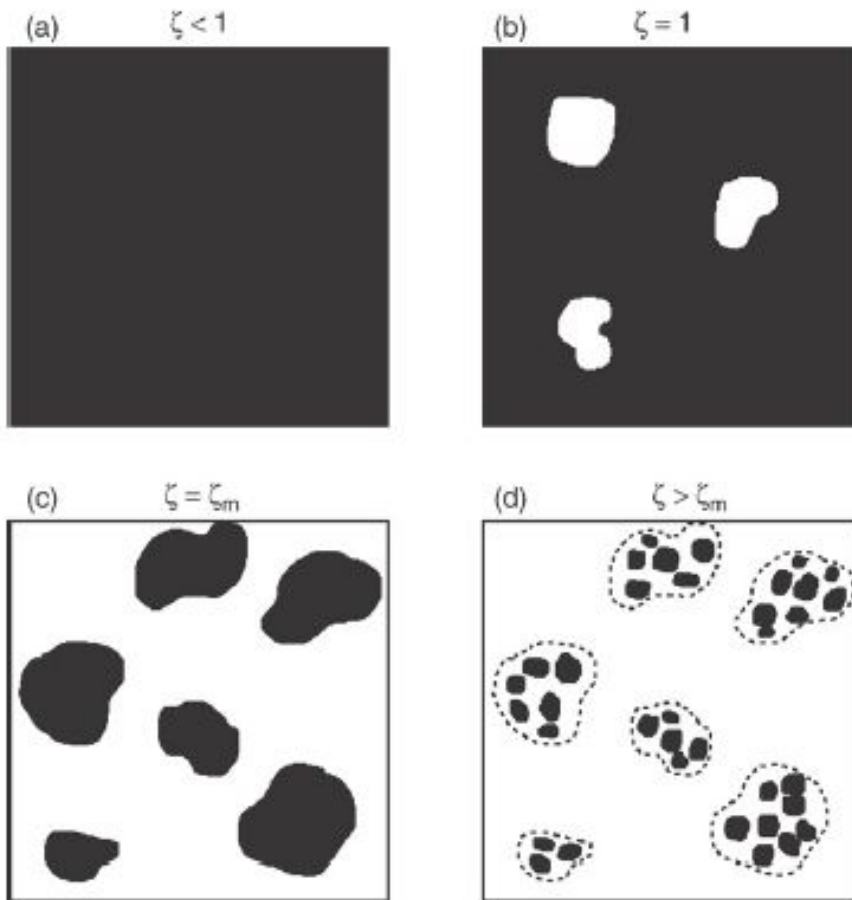


Fig. 12. The contact area at increasing magnification (a)–(d). The macroasperity contact area (c) breaks up into smaller contact areas (d) as the magnification is increased.

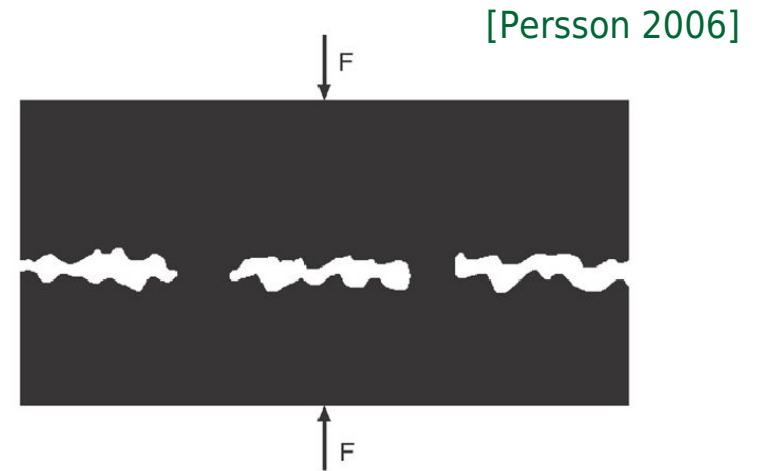


Fig. 1. Two solid blocks squeezed in contact with the force F . The area of real (atomic) contact A is usually an extremely small fraction of the nominal (or apparent) contact area A_0 .

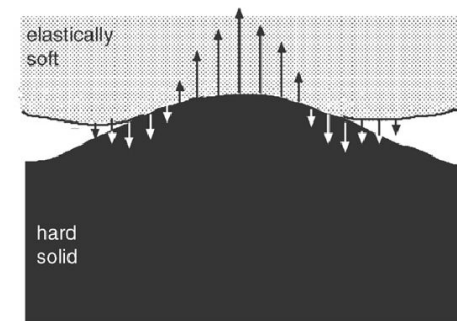


Fig. 17. The stress distribution in an asperity contact region. The stress is tensile close to the boundary line of the contact area. An attractive wall–wall interaction occurs also outside the area of real contact and this interaction determines the detachment stress σ_a and the work of adhesion γ .

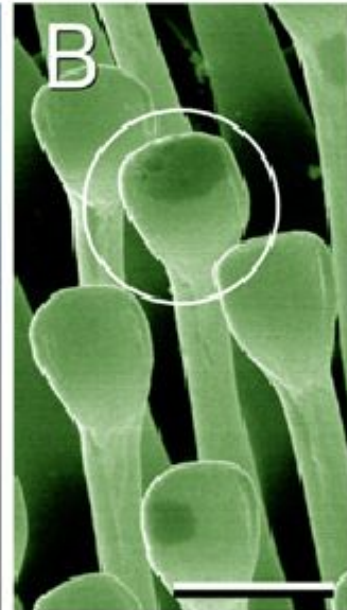
Off-the-point remark

[Persson 2006]

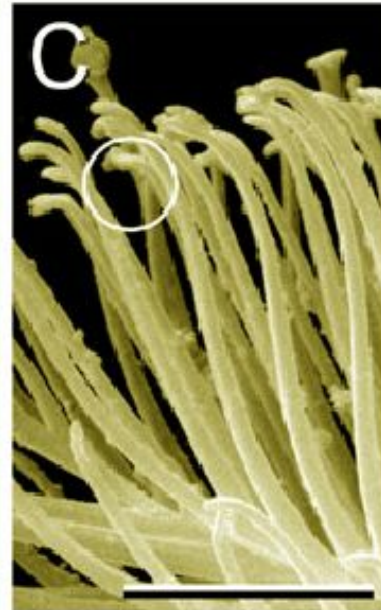
body mass →



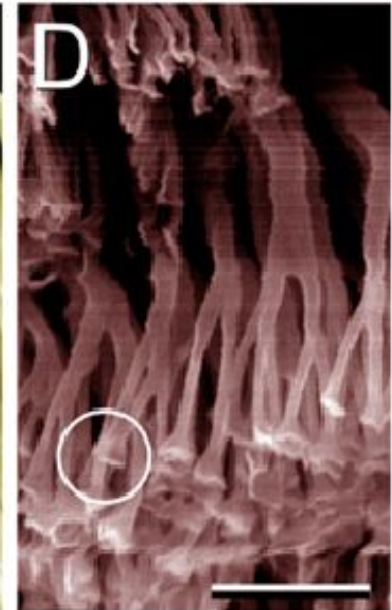
beetle



fly



spider



gecko

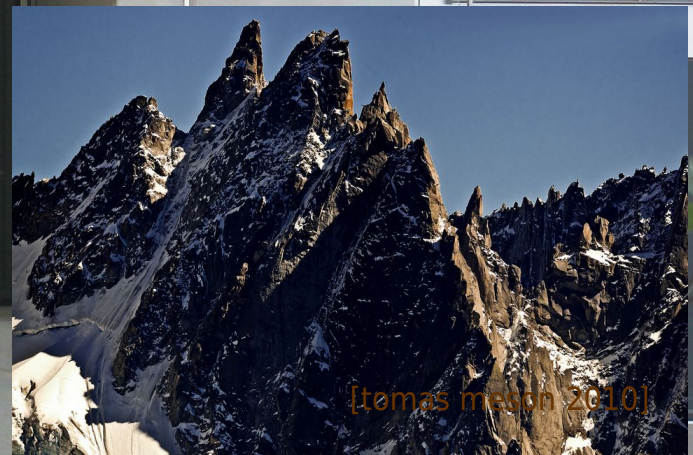
Concepts to keep in mind...

[lukax 2009]



Google for Britain's coastline, Benoît Mandelbrot, or fractals in nature :-)

- Fractals
- Fractal dimensions



[tomas meson 2010]

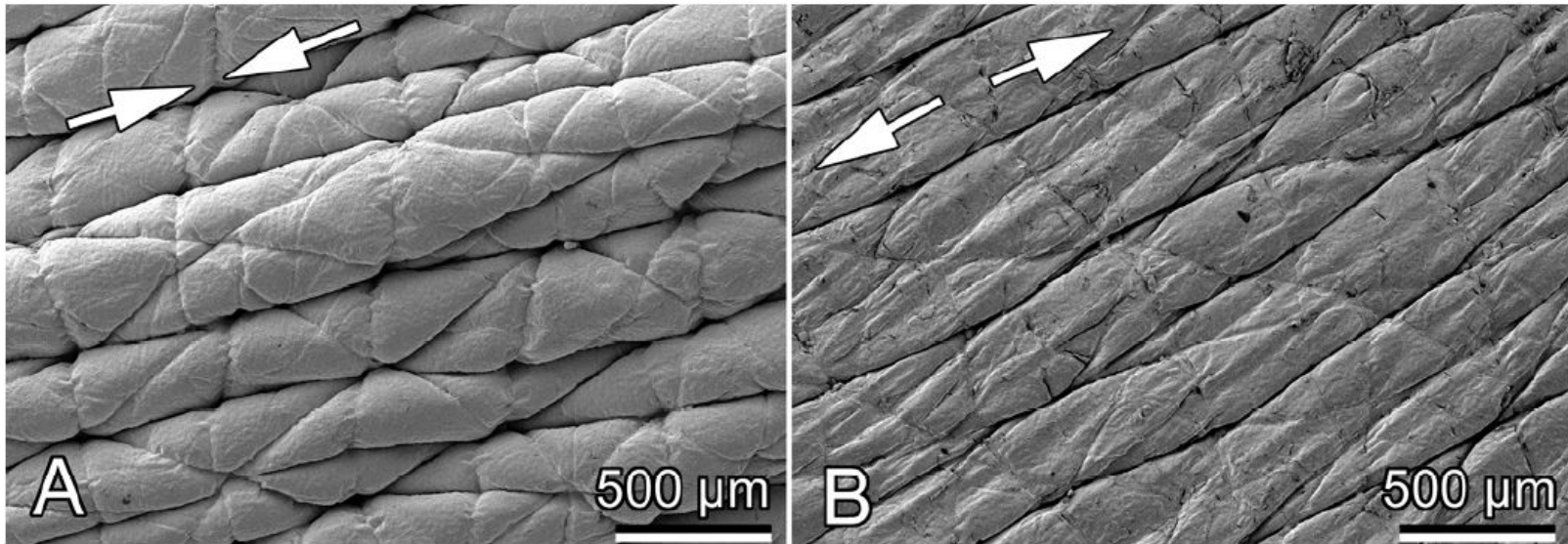
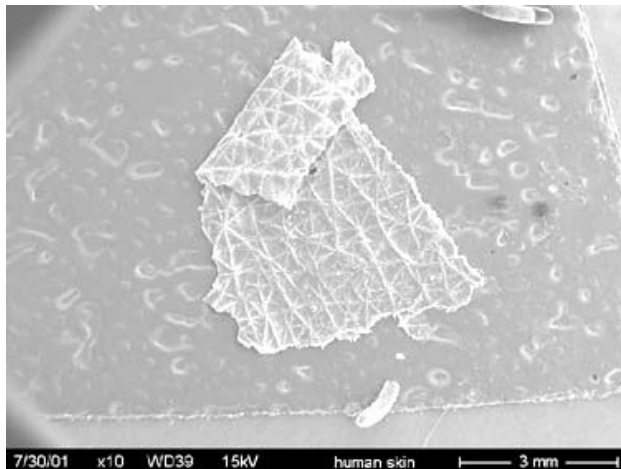


Figure 7. Use of the molding technique for visualisation of dynamic processes on the surfaces, which otherwise cannot be studied in the SEM. Pattern of the human wrist skin under compression (A) and under tension (B). Arrows show directions of compression and tension.



http://www.tedpella.com/replicat_html/44870.pdf

<http://www.physics.montana.edu/ical/gallery/SEM/Image%20of%20human%20skin.jpg>

Grease scatters light!



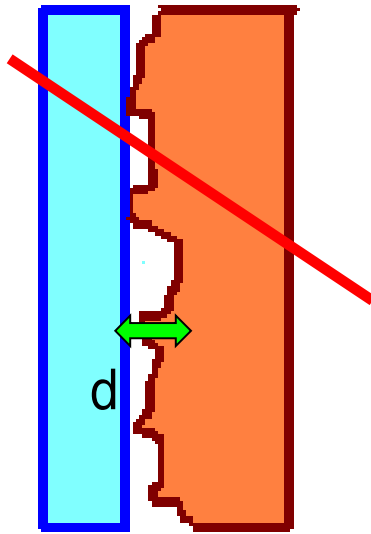
[Pink Sherbet Photography 2009]



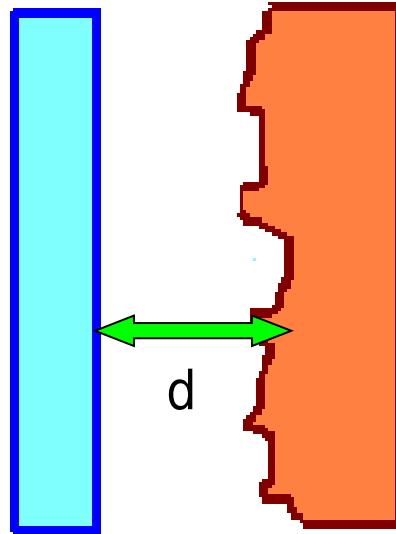
[Wolax 2009]

http://creativebits.org/files/images/dont_touch.png

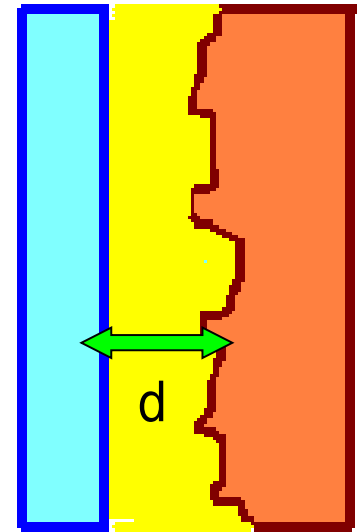
What touches glass?



if $d \ll \lambda \rightarrow$ direct glass-finger interface?
:-(seemingly, no



if $d \geq \lambda \rightarrow$ air gap,
optical interactions via
frustrated total internal
reflection?



if water/grease fills the gap
between "raised ridges" and
glass, is there a direct glass-
liquid interface?

- Noteworthy to investigate:

- washing and well drying hands vs greasy and/or wet hands?
- does the liquid scatter light or rather works as immersion fluid?
- are fingerprints visible when fingers are removed? (whatever yes or no, what does it mean?)

[Hiddenpower 2007]



No liquid: objects behind are visible



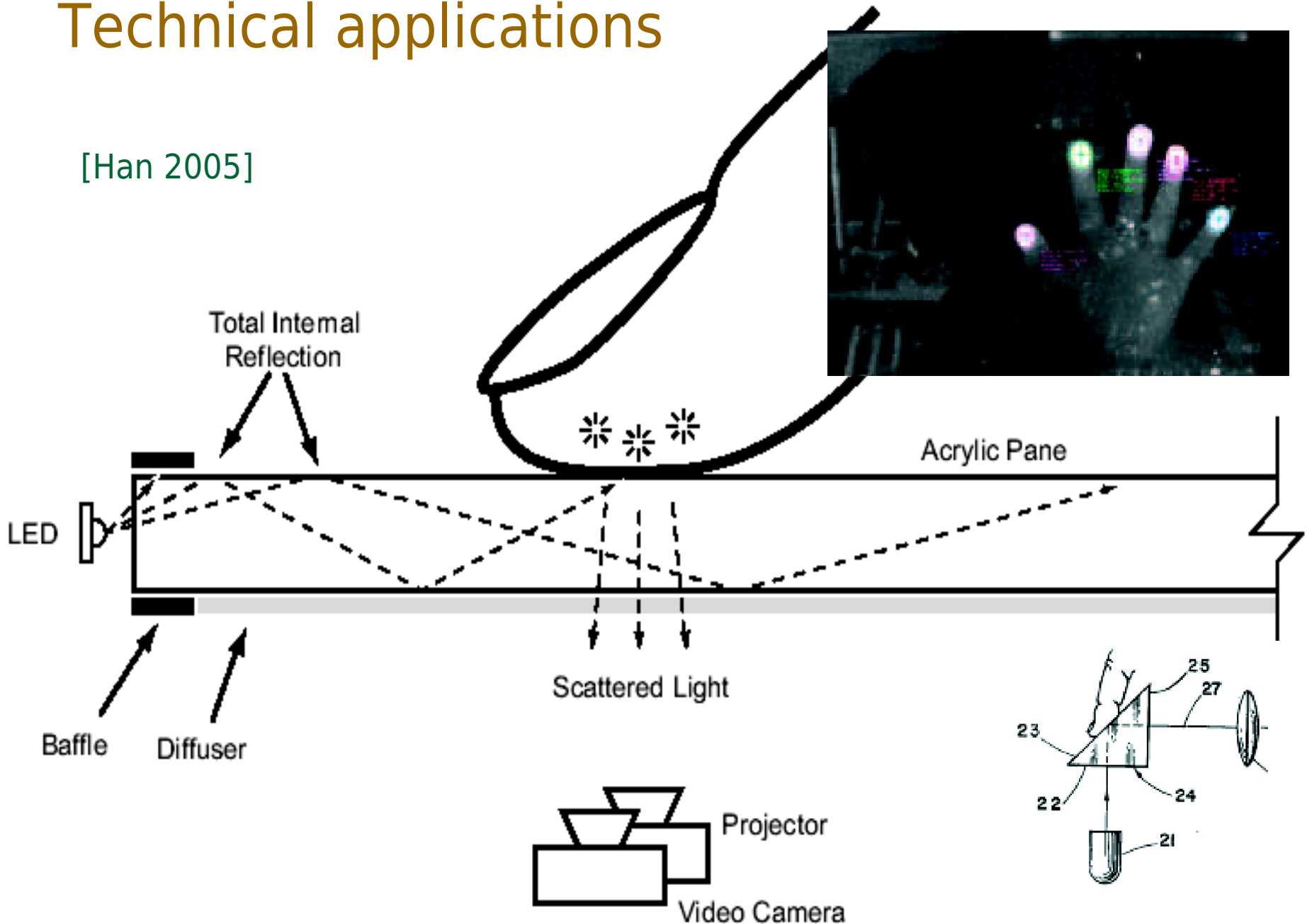
Liquid: objects behind are invisible

Would we see a laser pointing “into the camera”? (bright, directed light source?)

Would we see a dry napkin, a metal foil sheet, or even very dry fingers?
(what does it mean?)

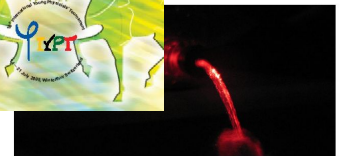
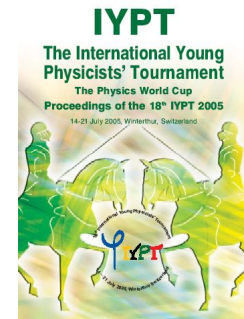
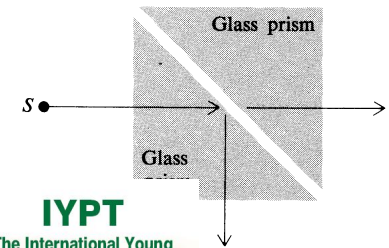
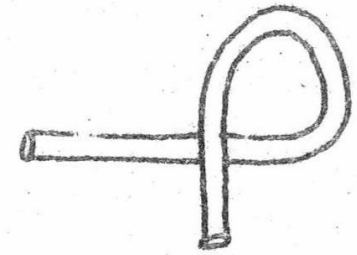
Technical applications

[Han 2005]



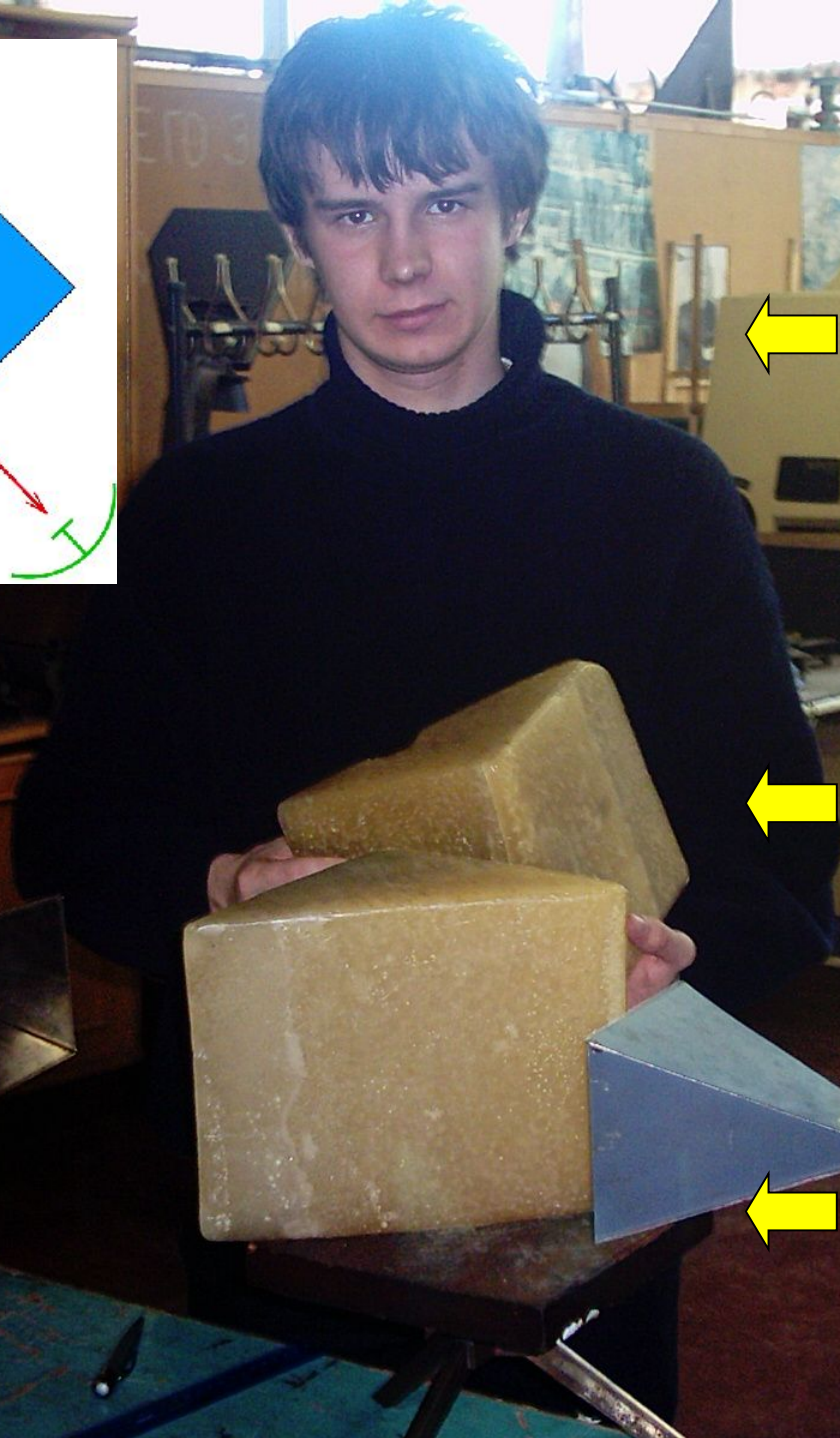
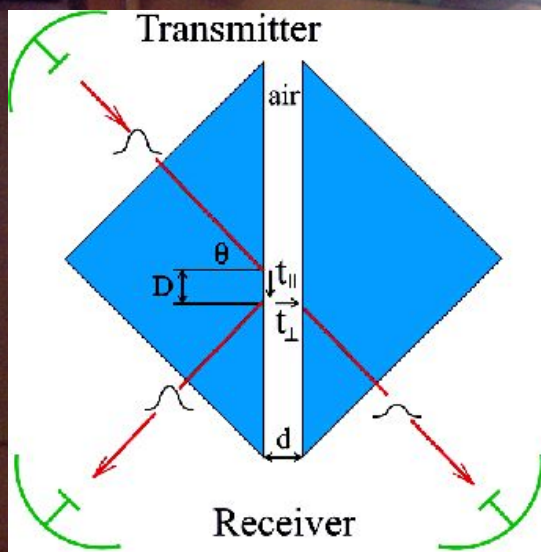
IYPT history

- **2. Light guide (3rd YPT, Correspondence Competition, 1981)**
 - The properties of light guides are well illustrated by a glass or a plexiglas rod, bent e.g. as shown in the picture. Study the properties of a similar, or a more interesting, light guide made in the school laboratory. Construct a device illustrating or using the properties of a light guide.
- **15. Optical tunneling (18th IYPT, 2005)**
 - Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally internally reflected.
- **8. Liquid light guide (23rd IYPT, 2010)**
 - A transparent vessel is filled with a liquid (e.g. water). A jet flows out of the vessel. A light source is placed so that a horizontal beam enters the liquid jet (see picture). Under what conditions does the jet operate like a light guide?



Problem No. 8 "Liquid light guide"

A transparent vessel is filled with a liquid (e.g. water). A jet flows out of the vessel. A light source is placed so that a horizontal beam enters the liquid jet (see picture). Under what conditions does the jet operate like a light guide?



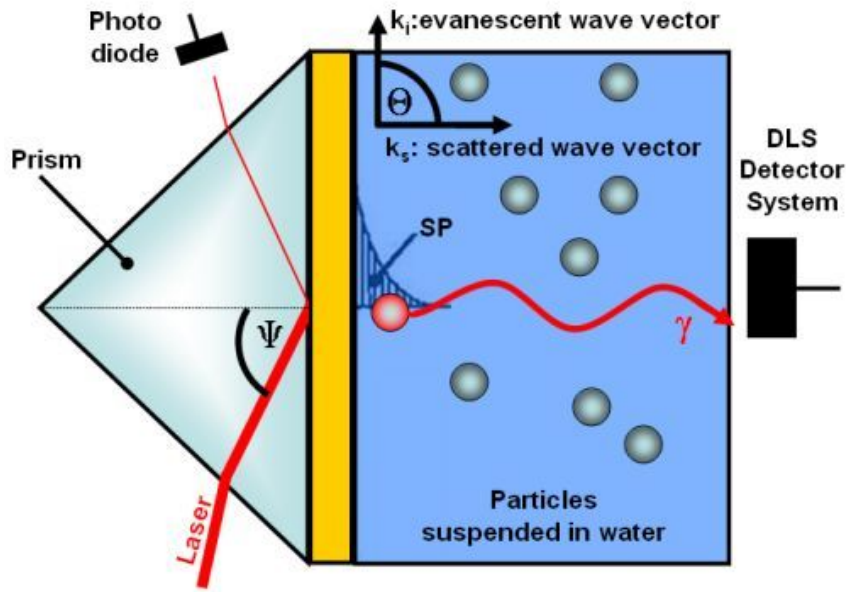
← Miša Valkovský

← Wax prisms as **model system** to investigate TIR with radio waves as a function of gap size, wavelength...

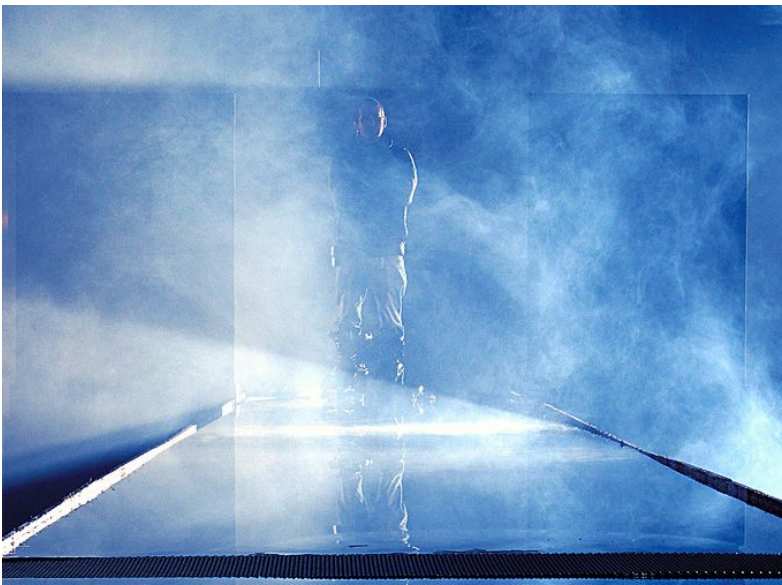
← Radio wave source, $\lambda = 3 \text{ cm}$

May 20, 2005

More visualization



- Particles just near the surface would scatter light in all directions, making “evanescent wave region” visible
- Smoke? Slightly turbid liquid?



Model system to look at light scattering from behind of the glass

But where to put the light source?

Hints and open questions

- What is the role of liquid in the effect? How does the image depend on its refractive index?
 - How to quantitatively characterize the dependence of visible pattern on the amount of liquid involved?
 - On what does the scattered light intensity (“brightness of fingerprints”) depend?
 - applied force → “effective contact area”?
 - observation angle and position of light sources?
 - amount of grease/water on fingertips?
 - ...
-

Key questions

- In the simplest case, if there are no fingers touching the glass, why there is nothing visible through the walls?
- Above all, what is the physical reason of the effect?
- What is the approximate gap between the skin and the glass for the raised epidermal ridges, and for the grooves between them? How to best approach the question, as the finger surface is fractal-like and gap varies considerably from point to point?
- What is the approximate length scale for roughness of the glass surface and of the finger skin? Is it physically correct to speak of such length scales? What other approaches would be more useful?
- How does the effect depend on skin properties, and is the finger grease relevant?
- How does the apparent contact area depend on length scale?
- What are the dependences of the reflective indexes in the system the angle of incidence? Are the Fresnel's relations relevant?
- Does the effect depend on wavelengths of incident or scattered light, refractive indexes of media in question, surface properties?
- Is it possible to establish a model experimental system to measure the radiation density at controlled distances from the interface? What range of wavelength would be optimum for such a system?
- Many approaches and concepts may emerge at discussions (momentum of photons, Pointing vector, tunneling, evanescent wave, potential barrier, probability distribution.) Can you discuss their relevance and re-formulate your explanation with a different basic concept?
- Is there a noteworthy time lag for the wave to pass through the gap?
- How to best record the visible fingerprint image for further analysis, and what information can be retrieved from such images?

Background reading (for courageous)

- Wikipedia: Total internal reflection, http://en.wikipedia.org/wiki/Total_internal_reflection
- Wikipedia: Evanescent wave, http://en.wikipedia.org/wiki/Evanescent_wave
- Wikipedia: Quantum tunneling, http://en.wikipedia.org/wiki/Quantum_tunneling
- B. N. J. Persson, S. Gorb. The effect of surface roughness on the adhesion of elastic plates with application to biological systems. J. Chem. Phys. 119, 21, 11437-11444 (2003), <http://juwel.fz-juelich.de:8080/dspace/bitstream/2128/1367/1/38088.pdf>
- B. N. J. Persson. Contact mechanics for randomly rough surfaces. Surface Sci. Rep. 61, 201-227 (2006), <http://www.multiscaleconsulting.com/resources/Contact+mechanics+for+randomly+rough+surface.pdf>
- Evanescent Waves (Carnegie Mellon University), <http://www.andrew.cmu.edu/user/dcprieve/Evanescent%20waves.htm>
- E. E. Hall. The penetration of totally reflected light into the rarer medium. Phys. Rev. 15, 73-106 (1902)
- D. D. Coon. Counting photons in the optical barrier penetration experiment. Am. J. Phys. 34, 240-243 (1965)
- J. C. Castro. Optical barrier penetration: A simple experimental arrangement. Am. J. Phys. 43, 107-108 (1974)
- A. I. Mahan and C. V. Bitterli. Total internal reflection: A deeper look. Opt. Soc. Am. 17, 509-519 (1978)
- S. Zhu, A. W. Yu, D. Hawley, and R. Roy. Frustrated total internal reflection: A demonstration and review. Am. J. Phys. 54, 7, 601-607 (1986), http://www.physics.princeton.edu/~mcdonald/examples/optics/zhu_ajp_54_601_86.pdf
- Seigo Igaki, Shin Eguchi, Fumio Yamagishi, Hiroyuki Ikeda, and Takefumi Inagaki. Real-time fingerprint sensor using a hologram. Appl. Optics 31, 11, 1794-1802 (1992), <http://www.xphotonics.com/tech/Finger%20Print/Real-time%20fingerprint%20sensor%20using%20a%20hologram.pdf>

Background reading (for courageous)

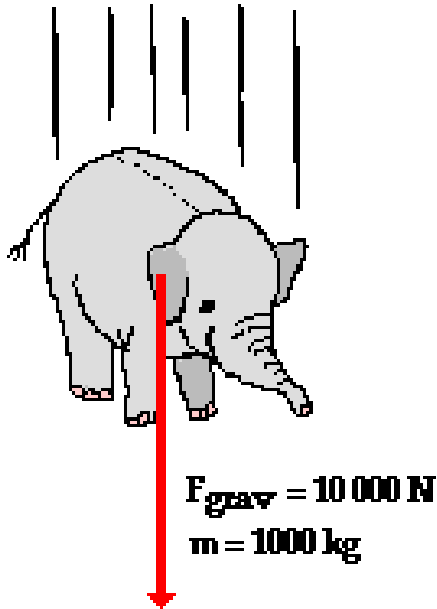
- Jefferson Y. Han. Low-cost multi-touch sensing through frustrated total internal reflection. Proc. UIST (Seattle, Oct. 23-27, 2005), pp. 115-118, <http://portal.acm.org/citation.cfm?id=1095054>
- Salvatore Esposito. Universal photonic tunneling time. Phys. Rev. E 64, 026609 (2001), [arXiv:physics/0102020v2](http://arxiv.org/abs/physics/0102020v2) [physics.optics]
- A. Haibel and G. Nimtz. Universal tunnelling time in photonic barriers. Ann. Phys. (Leipzig), 10, 8, 707-712 (2001), [arXiv:physics/0009044v1](http://arxiv.org/abs/physics/0009044v1) [physics.gen-ph]
- Yizhuang You, Xiaohan Wang, Sihui Wang, Yonghua Pan, and Jin Zhou. A new method to demonstrate frustrated total internal reflection. Am. J. Phys. 76, 3, 224-228 (2008), http://physlab.lums.edu.pk/images/6/6c/Frustrated_tir2.pdf
- G. Joos, I. M. Freeman. Theoretical Physics (Dover, New York, 1950)
- <http://books.google.com/books?id=duUJEp4WbQ8C&pg>
- Д. В. Сивухин. Курс общей физики. — М.: Наука, 1988. — т. 4, 5
- Луи де Бройль. Революция в физике. — М.: Госатомиздат, 1963
- Bruce W. Smith, Yongfa Fan, Jianming Zhou, Neal Lafferty, Andrew Estroff. Evanescent wave imaging in optical lithography. In: Optical Microlithography XIX, Proc. SPIE 6154, pp. 100-108 (2006), http://www.rit.edu/kgcoe/microsystems/lithography/research/imagetheory/SPIE_6154-10_smith-1.pdf
- Ignacy Gryczynski, Zygmunt Gryczynski, and Joseph R. Lakowicz. Two-photon excitation by the evanescent wave from total internal reflection. Anal. Biochem. 247, 69-76 (1997), <http://cfs.umbi.umd.edu/cfs/reprints/Two-Photon%20Excitation%20by%20the%20Evanescent%20Wave.pdf>
- A. A. Stahlhofen and G. Nimtz. Evanescent modes are virtual photons. Europhys. Lett. 76, 2, 189 (2006)



Problem No. 15 “Slow descent”

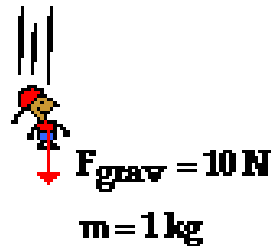
Design and make a device, using one sheet of A4 80 g/m² paper that will take the longest possible time to fall to the ground through a vertical distance of 2.5 m. A small amount of glue may be used. Investigate the influence of the relevant parameters.

(Very) basic ideas



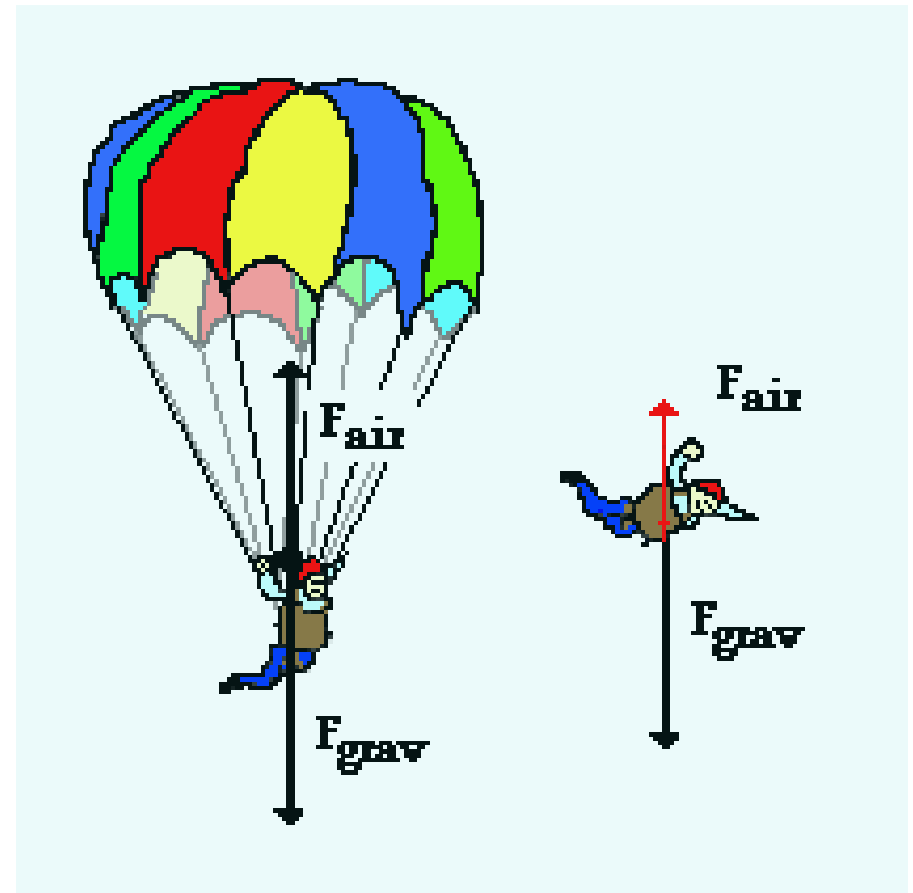
$$a = \frac{F_{net}}{m} = \frac{10\,000\text{ N}}{1000\text{ kg}}$$

$$a = 10\text{ m/s}^2$$



$$a = \frac{F_{net}}{m} = \frac{10\text{ N}}{1\text{ kg}}$$

$$a = 10\text{ m/s}^2$$

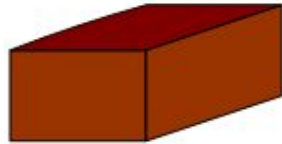


Vacuum, no air drag $\rightarrow a \neq f(\text{mass, shape})$

Air drag $\rightarrow a \neq f(\text{mass}), a = f(\text{shape})$



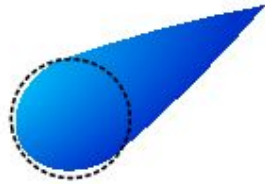
Low drag coefficient.



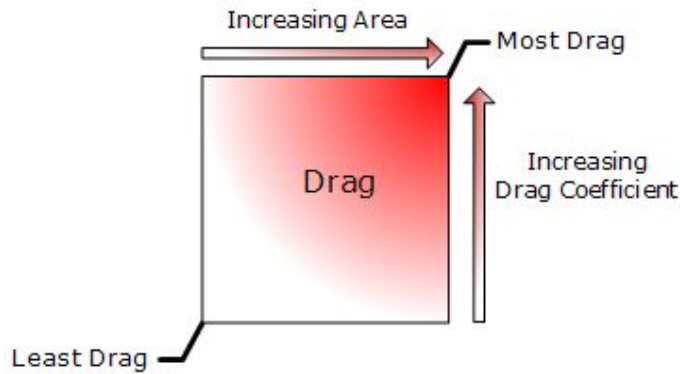
High drag coefficient.



Low frontal area.



High frontal area.



The force of drag increases with a higher drag coefficient and from a larger frontal area.

What is drag?

- Aerodynamic resistance force, not related (directly) to viscosity
- In the simplest case, implies that the motion is just translational and stationary

- The mass of the air that hits the object per time dt :

$$dm = v \cdot \rho \cdot S \cdot dt$$










Flow considered as elastic particles colliding the object → maximum transferred momentum is:

$$dp = 2v \cdot dm$$

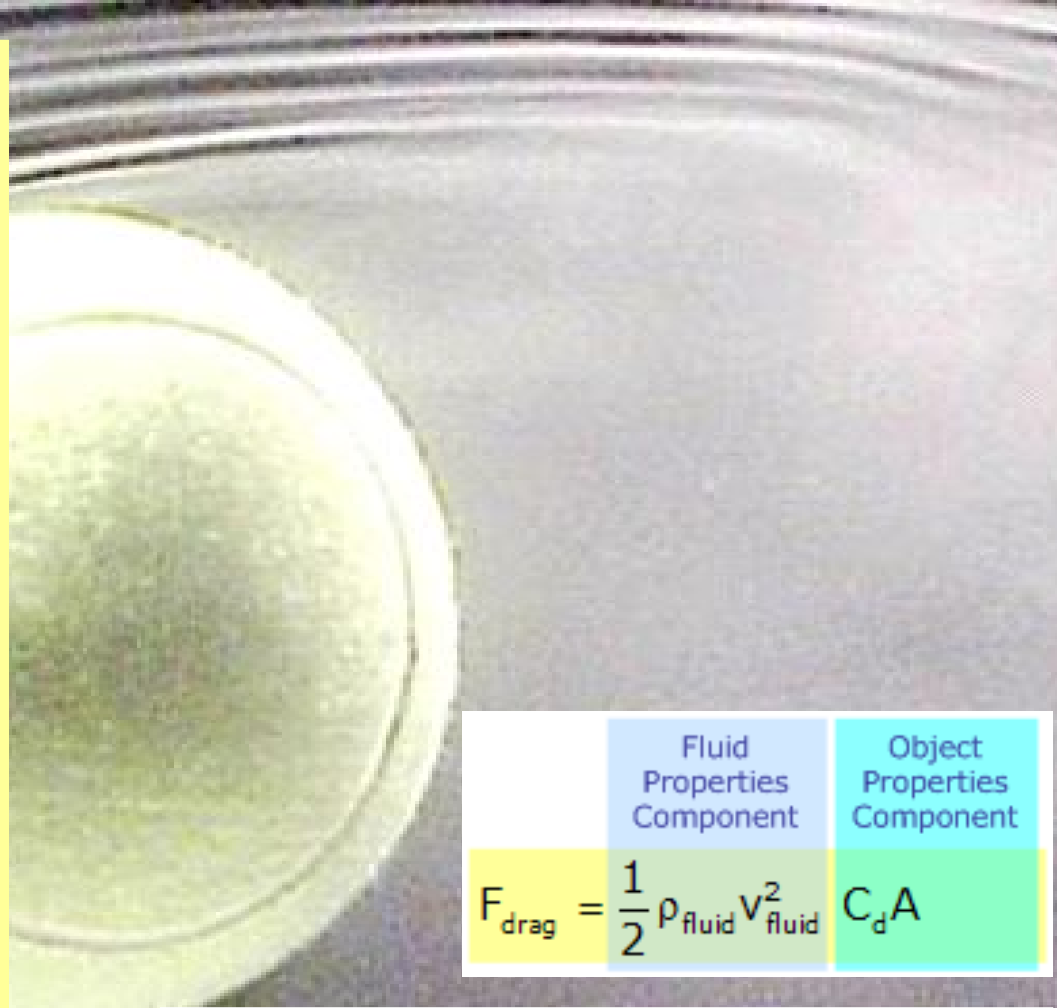
- Energy losses may be considered as a factor C :

$$dp = C v \cdot dm$$

$$F = \frac{dp}{dt} \quad \Rightarrow \quad F = C \cdot v \cdot \frac{dm}{dt} = C \rho v^2 S$$

Shape	Drag Coefficient
Sphere → 	0.47
Halfsphere → 	0.42
Cone → 	0.50
Cube → 	1.05
Angled Cube → 	0.80
Long Cylinder → 	0.82
Short Cylinder → 	1.15
Streamlined Body → 	0.04
Streamlined Halfbody → 	0.09

Measured Drag Coefficients



	Fluid Properties Component	Object Properties Component
$F_{\text{drag}} =$	$\frac{1}{2} \rho_{\text{fluid}} v_{\text{fluid}}^2$	$C_d A$

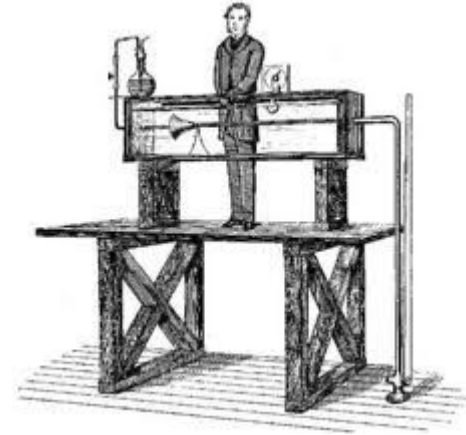
- ρ_{fluid} is the mass density of the fluid
- v_{fluid} is the velocity of the object relative to the fluid
- A is the reference area
- C_d is the drag coefficient (dimensionless constant)

Reynolds number

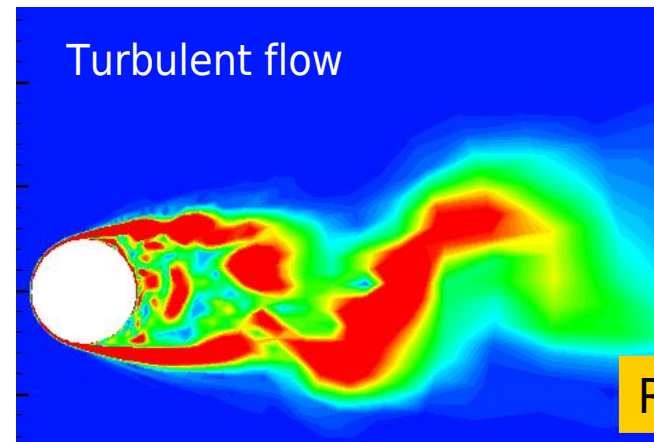
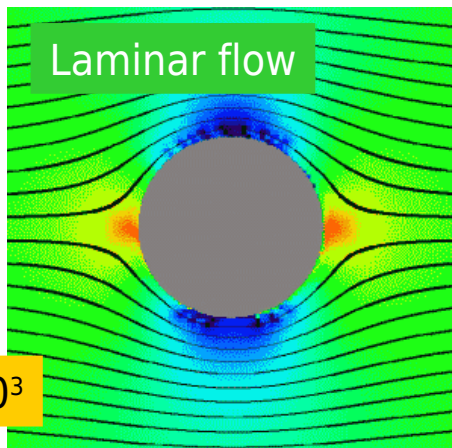
- Reynolds number characterize the ratio between inertial and viscous forces for a fluid flow,

$$Re = \frac{\rho_{fluid} v_{fluid} d}{\eta}$$

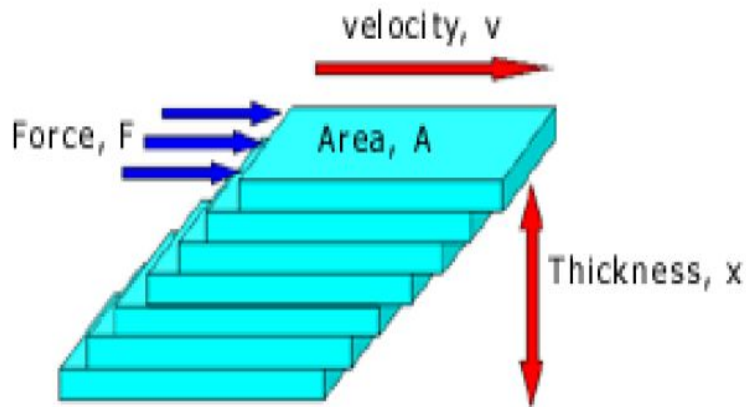
- ρ_{fluid} is the mass density of the fluid
- v_{fluid} is the velocity of the object relative to the fluid
- d is the characteristic length
- η is the dynamic viscosity [Ns/m^2]



- Related to the probability of laminar-turbulent transition



What is viscosity?



Shear stress [Pa]:

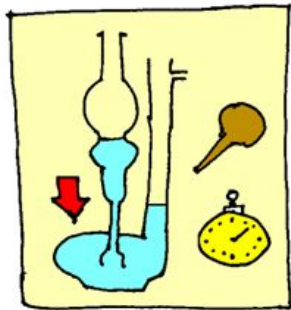
$$\tau = \frac{F}{A}$$

Shear rate [s^{-1}]:

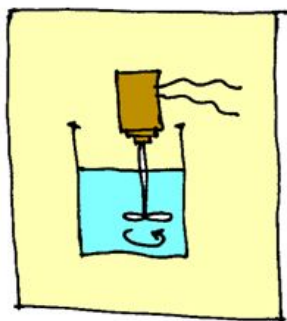
$$g = \frac{v}{x}$$

Viscosity (at definite moment) [Pa*s]:

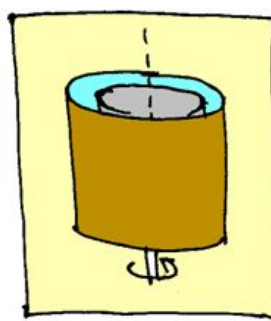
$$\eta = \frac{\tau}{g}$$



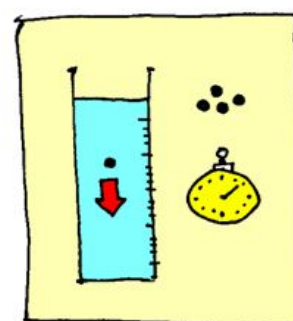
Ostwald's viscometer



Rotational viscometer



Coaxial cylinders



Stokes' method

Nice to be well familiar with, but is viscosity of much relevance for the task?

Free fall: terminal velocity

- Simplest approximation: force is proportional to the instant speed (**but is it always the case?**):

$$\mathbf{F} = -\alpha\mathbf{v}.$$

- Equation of motion:

$$m\frac{d\mathbf{v}}{dt} = \mathbf{F}_g + \mathbf{F}_d = m\mathbf{g} - \alpha\mathbf{v}$$

- The drag grows as the speed grows; the body **deccelerates**:

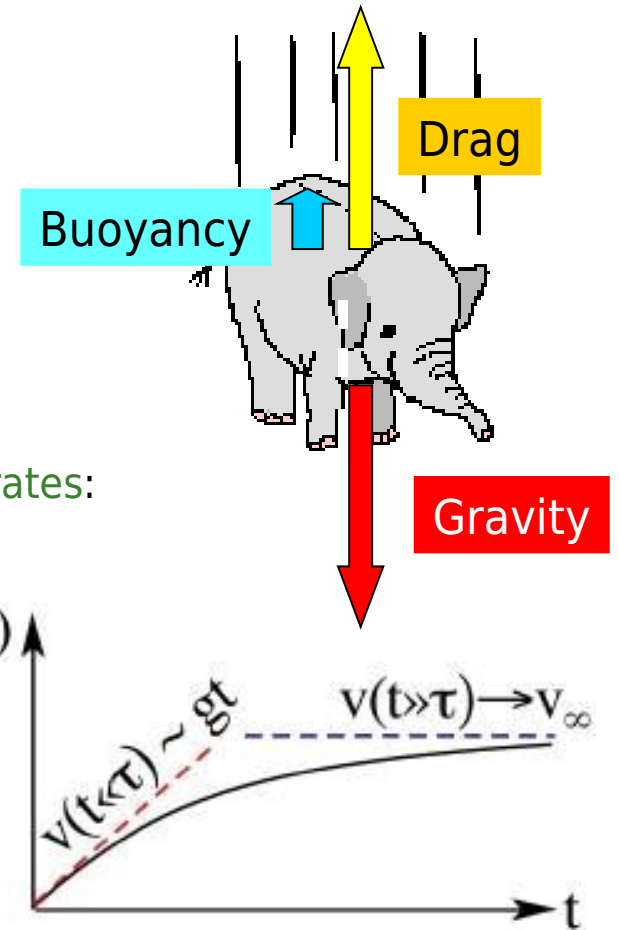
$$m\frac{d\mathbf{v}}{dt} \approx \mathbf{F}_g + \mathbf{F}_d = m\mathbf{g} - \alpha\mathbf{v} \Rightarrow v_\infty = \frac{mg}{\alpha}.$$

- What is the time dependence for speed in the transitory regime?

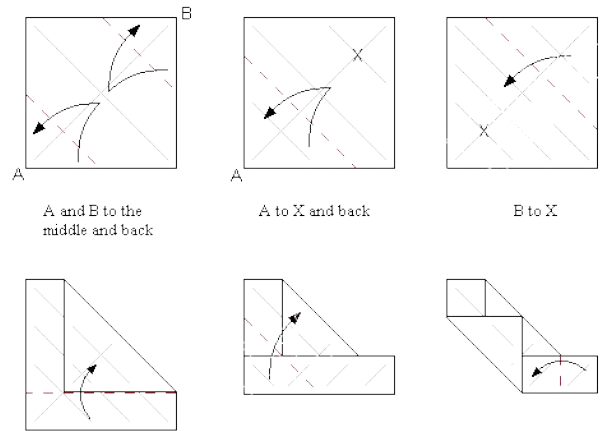
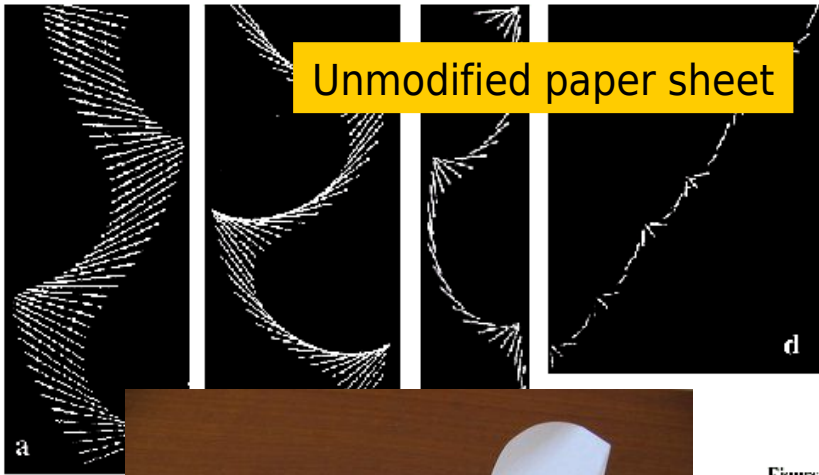
$$m\frac{dv_z}{dt} = mg - \frac{1}{2}C_dA\rho v_z^2.$$

- Separation of variables and integration leads to:

$$\int_{v=0}^v m \frac{dv_z}{mg - \frac{1}{2}C_dA\rho v^2} = \int_{t=0}^t dt. \quad \Rightarrow \quad v(t) = v_\infty \tanh(t/\tau), \quad v_\infty = \sqrt{\frac{2gm}{AC_d\rho}}, \quad \tau = v_\infty/g.$$



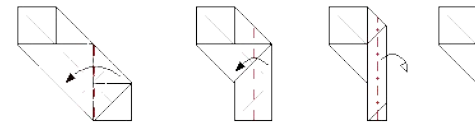
Possible ideas for a device...



Paper airglider



Figure 1



Paper maple seed

Paper propeller



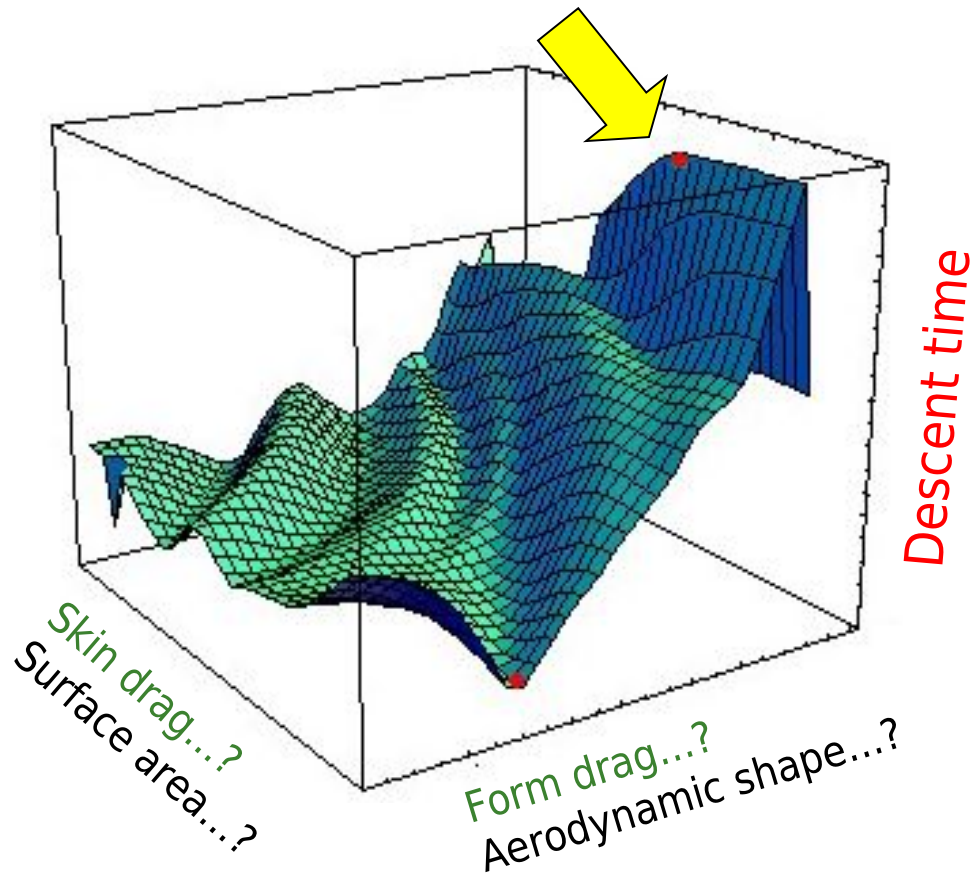
Hey, the problem is in physics and not in checking all possible engineering concepts!



Paper parachute

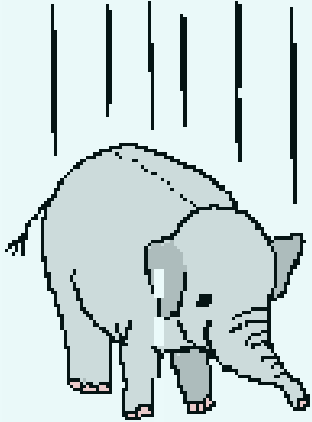


Maximizing the descent time...

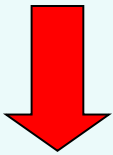


- We are looking for a global extremum of a function with multiple variables
- Some are rather fixed, some are not
- Amount of material (fixed)
- Initial height (fixed)
- Air density and viscosity (fixed)
- Ambient airflows (wind), not induced by the device itself (fixed as zero?)
- Surface properties (fixed or not?)
- Aerodynamic shape (not fixed)
- Surface area (not fixed)
- Linear dimensions, e.g. cross-section (not fixed)

Energy conservation approach...?



$$U = mgh$$
$$mv^2/2 = 0$$
$$I\omega^2/2 = 0$$



That means the
minimum descent time!

~~$$U = 0$$
$$mv^2/2 = mgh$$
$$I\omega^2/2 = 0$$~~

$$U = 0$$
$$mv^2/2 \rightarrow 0 \quad :-)$$

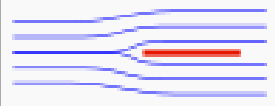
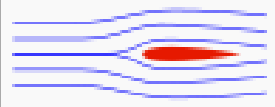
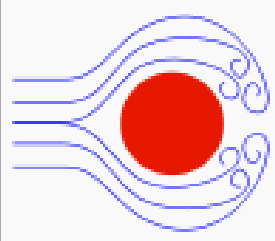
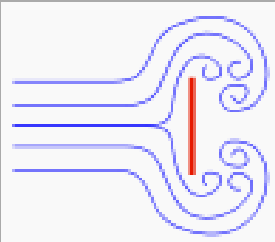


How to achieve it?



How to maximize the descent time...?

- Make the device transferring its initial potential energy into anything but kinetic energy of vertical descent?
 - into kinetic energy of rotational motion? (rotating propeller?)
 - into kinetic energy of horizontal motion? (glider?)
 - into kinetic energy of airflow or into fluid friction? (specific shape to induce turbulence, higher air drag?)
 - into vibrations, or elsewhere? (any ideas how?)
- Make the device experiencing higher air resistance?
 - form drag → adjusting shape and size
 - skin drag → adjusting surface area
- What about the stiffness of the entire construction?
 - can internal motion and friction in a non-rigid body be helpful?

Shape and flow	Form drag	Skin friction
	0%	100%
	~10%	~90%
	~90%	~10%
	100%	0%

To keep in mind...

- Is the drag **always proportional** to translational speed? What about rotational speed, if the device rotates?
 - Is the **air density constant** in all points around the falling body?
 - How does the aerodynamic behavior depends on Reynolds' number? Is the **Reynolds number constant** over time of flight?
 - Is the **air viscosity** not at all relevant?
 - How do the translational and angular speeds and accelerations for your device depend on time?
 - Is there any regularity in the **spatial orientation** of the device during falling?
 - What features of the **device's geometry** make it fall as it falls?
 - From the energy point of view, what is the initial potential energy (mgh) in comparison to translational or rotational kinetic energies during different stages of the flight?
-

Paper propeller...?

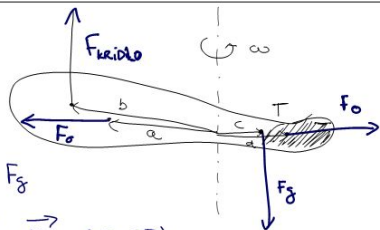


Problem No. 2 "Winged seed"

Investigate the motion of falling winged seeds such as those of the maple tree.

- See the reference kit from 2008

Anályza síl (sústava semienka)



$$F_{\text{keřidlo}} = F_g$$

$$\vec{F}_{\text{keřidlo}} \times \vec{b} + \vec{F}_0 \times (\vec{\omega} + \vec{\mathcal{R}}) + \vec{F}_g \times \vec{c} = \vec{0}$$

11. okt6bra 2010

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- See Martin's slides from 2008

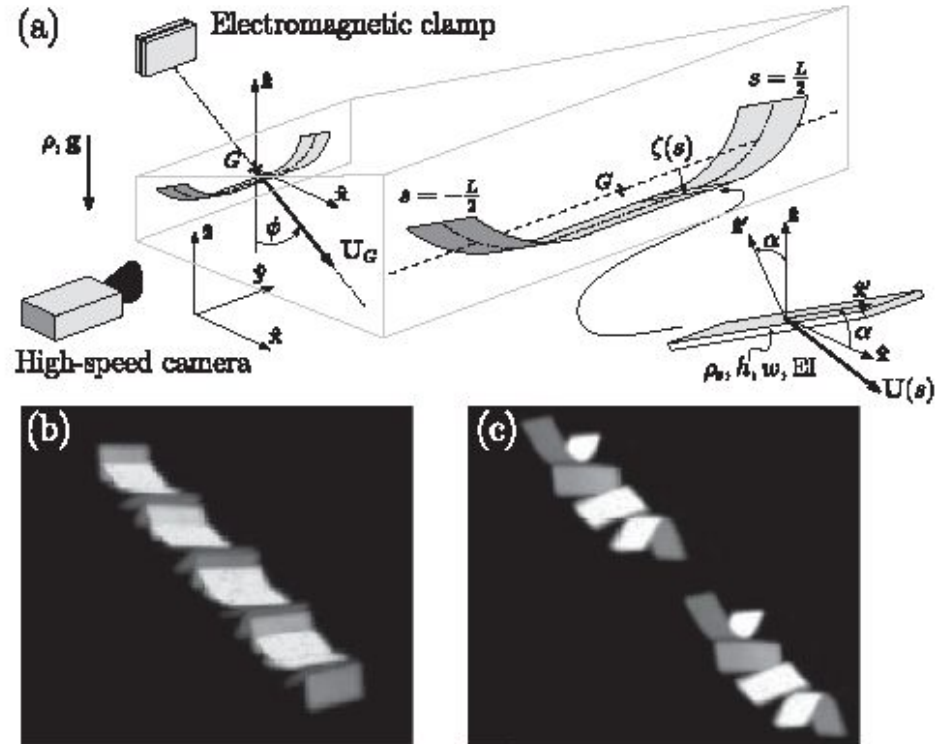
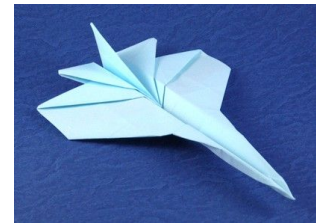
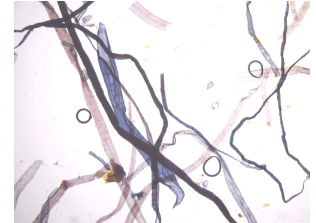


FIG. 1. (a) Schematic of the geometry of a falling, tumbling, bent wing. Superimposed snapshots of a tumbling paper wing: short wings remain straight (b), while long wings bend (c). For the sake of clarity, two half cycles are shown in (c) and images are artificially spaced. (See videos [11].)

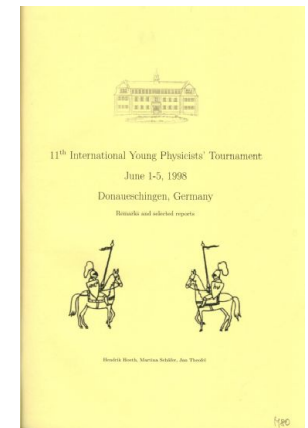
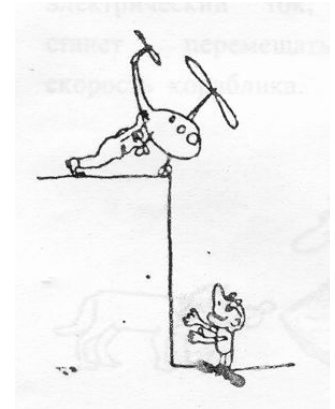
Tricky points: interpretation of the task

- Can we use only a fraction of the A4 sheet?
 - A tiny paper particle will descend for the longest time, and a nanoscale paper particle will never descend due to Brownian motion. But are we violating the task?
- If no, can we make a highly porous device by grinding the entire A4 sheet into dry fibers and stabilizing it with an aerosole of “a small amount of glue” (“dandelion”)
 - Larger surface area means higher air drag. But are we violating the task?
- Can we focus efforts on a glider or a paper plane?
 - The vertical speed (descent rate) will be quite small, but can we say this motion is “falling”?
- Certain devices may require a particular “lauch procedure” (to establish a good angle of attack, to gain an initial torque, or a spatial orientation)
 - Do we violate the task by imposing any of such conditions?
- Finally, what about making a hot-air paper balloon with “a small amount of glue” as fuel?
 - It may look like a forbidden trick, but if other teams do so, how to correctly oppose their approach?



IYPT history

- **9. Passive motor (4th IYPT, 1991)**
 - An apple dropped from a balcony of a multi-storey building will slowly descend into the hands of your friend if a propeller cut out of rigid paper is attached to this apple with a match. Explain the principle of work for such a parachute and study the dependence of the drag force on the speed of falling and on the sizes of the propeller's blades.
- **1. Invent yourself (11th IYPT, 1998)**
 - Construct an aeroplane from a sheet of paper (A4, 80 g/m²). Make it fly as far and/or as long as possible. Explain why it was impossible to reach a greater distance or a longer time.
- **5. Dropped paper (12th IYPT, 1999)**
 - If a rectangular piece of paper is dropped from a height of a couple of meters, it will rotate around its long axis whilst sliding down at a certain angle. How does this angle depend on various parameters?
- **2. Winged seeds (21st IYPT, 2008)**
 - Investigate the motion of falling winged seeds such as those of the maple tree.



Problem No. 2 "Winged seed"
Investigate the motion of falling winged seeds such as those of the maple tree.

Key questions

- What physical parameters determine the angle of attack and the spatial orientation of the falling paper device?
 - What kind of motion is preferential for the device to maximize the descent time? (stable rotational motion? gliding? translational motion?)
 - What are the magnitudes of the Reynolds number for the flow around the device? Is the flow laminar or turbulent? Does the Reynolds number change with time?
 - How to visualize the flow around the falling device?
 - How fast would it descend and fast would it move along a horizontal axis?
 - If your device shows a maximum descent time from the height of 2.5 m, would it be suitable for smaller or larger heights? What is the dependence of descent rate on time for your device?
-

Background reading (for courageous)

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[conskeptical 2008]

Problem No. 4 “Breaking spaghetti”

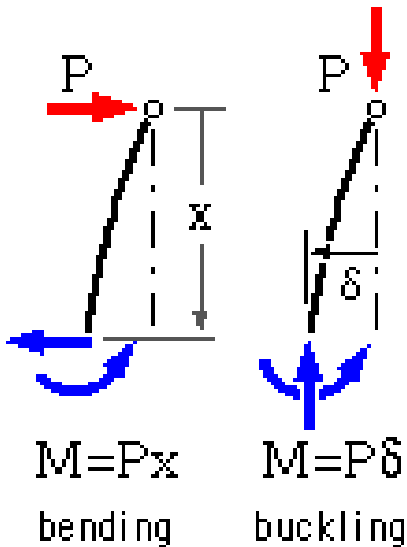
Find the conditions under which dry spaghetti falling on a hard floor does not break.



Curved spaghetti are likely to break into multiple pieces →
2006 Ig Nobel Prize in Physics awarded to Sébastien Neukirch and Basile Audoly

Spaghetti hits the floor...

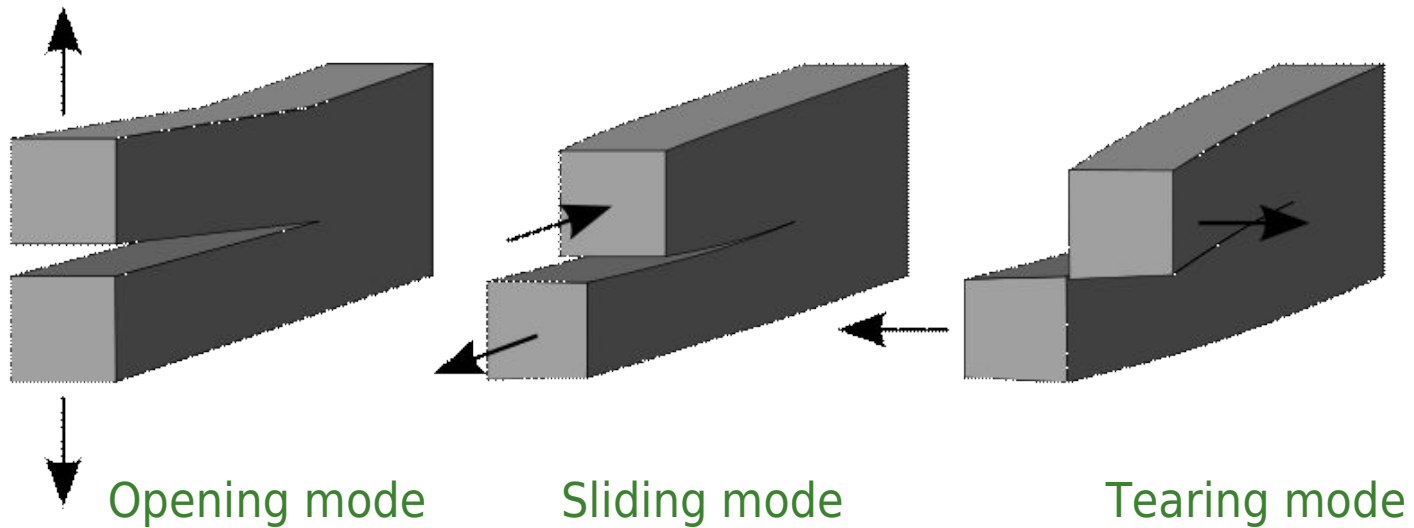
- vertically...?
- horizontally...?
- inclined, under a certain angle...?
- For what time is the spaghetti subject to impact load, and how do the stresses depend on time, in different points?



Bending deformation is usually proportional to the load

Buckling is a displacement of a structure (subjected usually to compression) **transverse to the load**. Moments, deflections and stresses are **not proportional** to loads

Fracture modes in spaghetti...



<http://www.math.psu.edu/belmonte/sg9.gif>

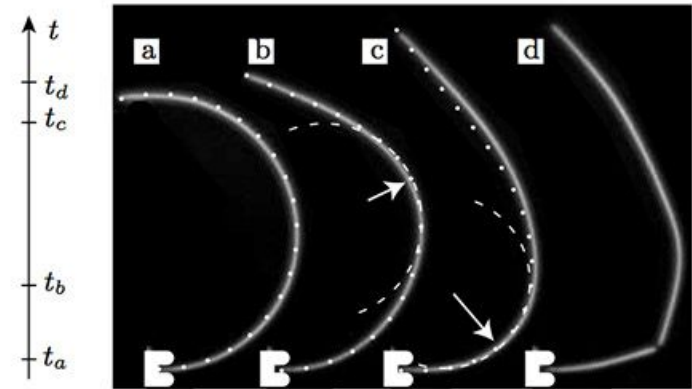
J.R. Gladden & A. Belmonte, Penn State (2003)

- Longitudinal compression causes buckling

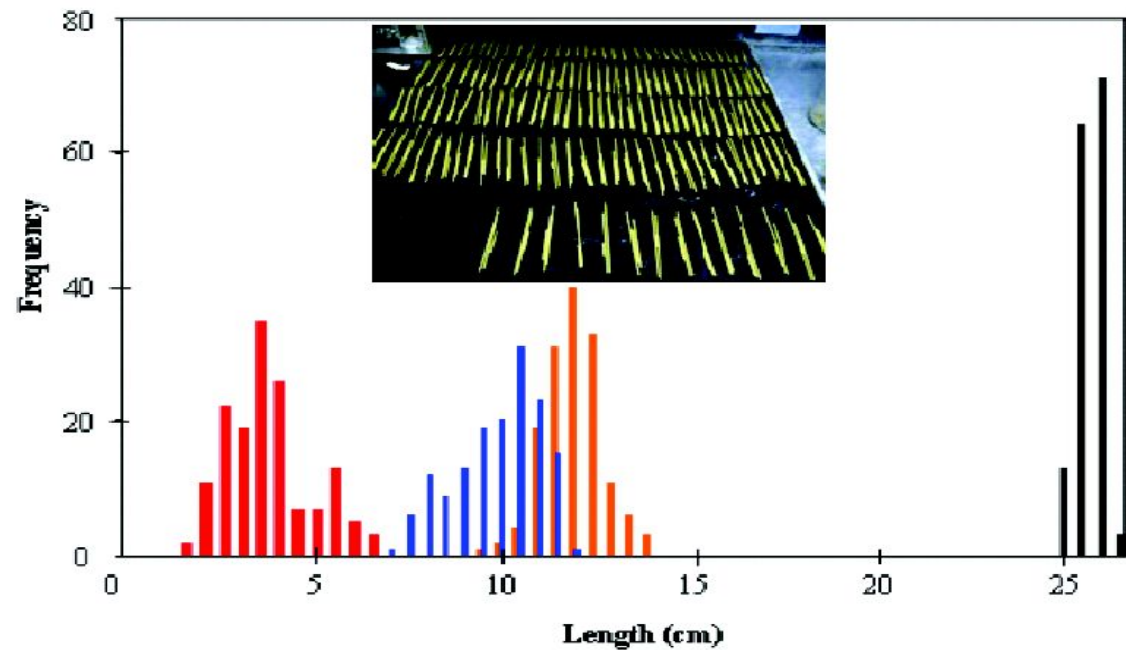


Fracture of bent spaghetti...

- Curvature before fracture



- Size distributions for spaghetti debris



Chances to break...?



- Harder floor or more fragile spaghetti?
- Different impact speed, angular velocity?
- Different impact angles?
- A single rod or bulk?

- How about posing a reverse problem: what information about the impact can be retrieved from the spaghetti debris?

Is it worth speaking of a **probability of fracture**, under given conditions? Is it possible to measure and/or theoretically predict it?

“Final optimization”

- What is the safest impact speed? (less seems better :-))
 - What is the safest impact angle? (why? what maximum stresses are expected for different impact angles?)
 - What are the mechanical properties for a floor to be still considered “hard”?
 - What is the minimum safe curvature for a spaghetti to withstand?
 - Above all, if all conditions are fulfilled and all parameters are optimized, what is the probability of fracture?
-

Open questions

- If a spaghetti falls with a random spatial orientation, what is the **probability of fracture**? Is there a way to determine the probability experimentally?
- Unlike the well-studied phenomenon of spaghetti breaking under gradual, controlled bending, we have an **instant impact stress**. What is common and what is different in these two situations? What is the time lag between initial contact and final rupture? What are the time scales for deformation and fracture?
- How to **measure, control or predict** the impact and fracture dynamics?
- How to best record the process (with **high-speed camera**?)
- What is the role of the friction between the spaghetti and the floor in case of non-horizontal orientation at impact? Are surface roughnesses relevant?
- How hard is a “hard” floor? What of its **parameters** can be controlled?
- How to measure the **mechanical properties** of the spaghetti you work with? How do these properties vary among different brands?
- At what degree the effect is **reproducible**, if the experiment is repeated under “identical” conditions?

Background reading (for courageous)

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<http://www.tinkerhack.com/photos/highres/OrganizationsAndPlaces/Exploratorium/WithRobertAllen/Wot/ShiftingVibratingSandPile.jpg>



Problem No. 10 “Faraday heaping”

When a container filled with small spheres (e.g. mustard seeds) is vibrated vertically with a frequency between 1 - 10 Hz, so called Faraday heaping occurs. Explore this phenomenon.

XVII. *On a peculiar class of Acoustical Figures; and on certain Forms assumed by groups of particles upon vibrating elastic Surfaces.* By M. FARADAY, F.R.S. M.R.I., *Corr. Mem. Royal Acad. Sciences of Paris, &c. &c.*

Read May 12, 1831.

1. **T**HE beautiful series of forms assumed by sand, filings, or other grains, when lying upon vibrating plates, discovered and developed by CHLADNI, are so striking as to be recalled to the minds of those who have seen them by the slightest reference. They indicate the quiescent parts of the plates, and visibly figure out what are called the nodal lines.

2. Afterwards M. CHLADNI observed that shavings from the harp vibrating violin bow did not proceed to the nodal lines, but were gathered on those parts of the plate the most violently agitated, i. e. at the points of greatest oscillation. Thus when a square plate of glass held horizontally above and below at the centre, and made to vibrate by the application of a violin bow to the middle of one edge, so as to produce the loudest sound, sand sprinkled on the plate assumed the form of a diamond; but the light shavings were gathered together at those parts towards which the vibrations of the four portions where the vibrations were most powerful and the oscillations of the plate greatest.

3. Many other substances exhibited the same appearance. Lycopodium,

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TRANSACTIONS
OF THE
ROYAL SOCIETY
OF
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MDCCCXXXI.



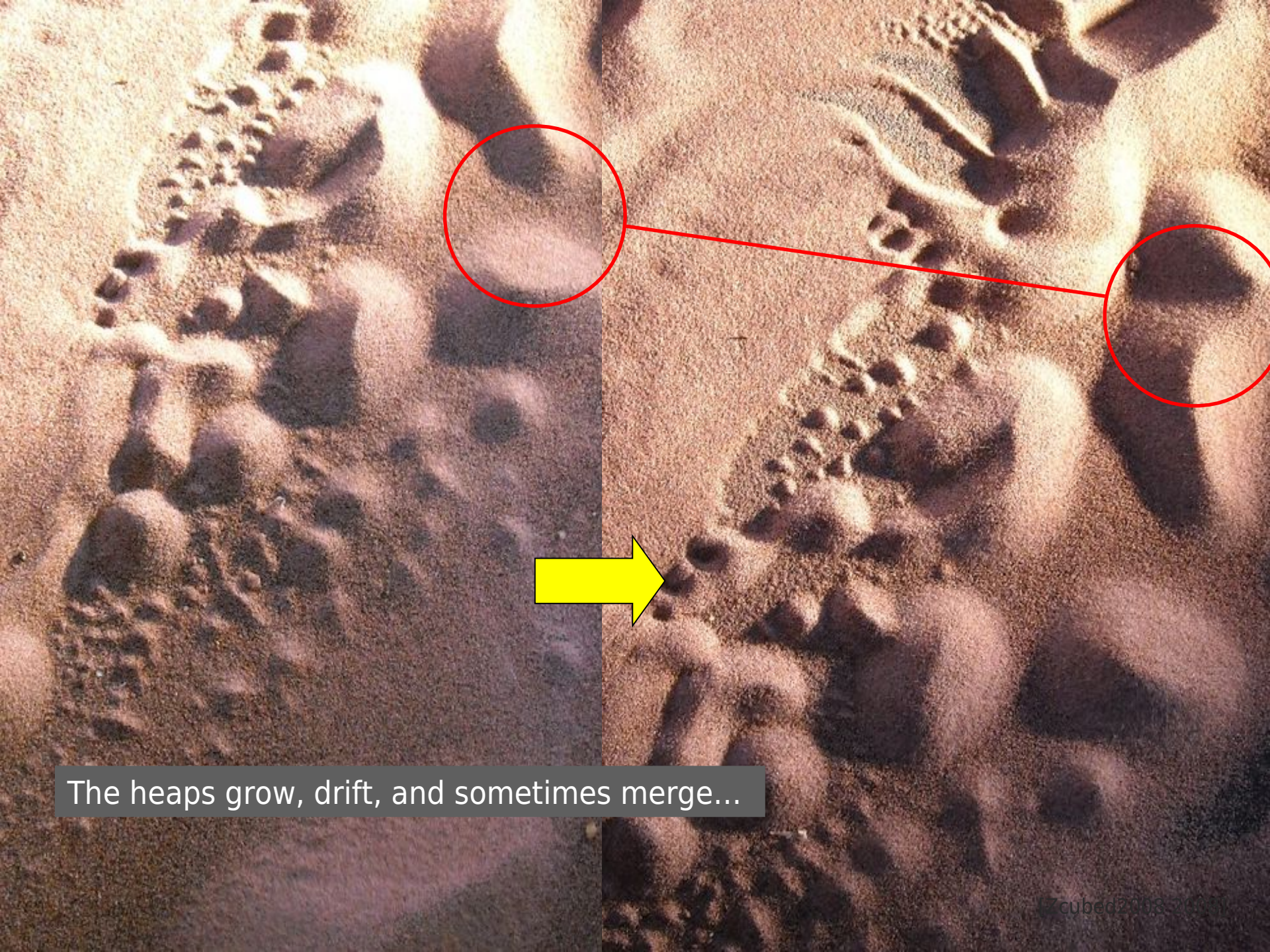
Chladni figures →
Nice, but not exactly our problem :-)



Heaps and ripples appear on different length scales...



Granules are not packed, and air seemingly plays a role in the effect...



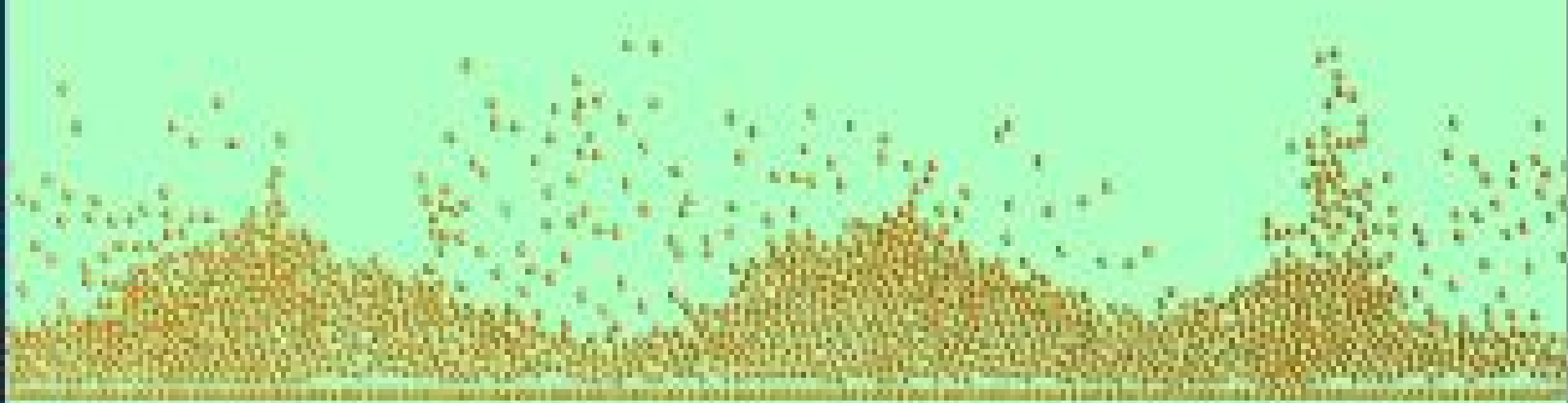
The heaps grow, drift, and sometimes merge...

[adamrossbarker 2007]



Parameters of shaking determine the overall pattern...

http://www.ph.biu.ac.il/~rapaport/anim_gif/vibgran_anim.gif



spontaneous pattern formation in a dynamical system outside of equilibrium [4].

The dynamic stabilization of a single heap (last snapshot of Fig. 1) is well understood: The outward avalanches in the upper layers are balanced by the inward motion of the deeper layers (induced by the airflow through the vibrating bed [2,3]), together forming the convective flow of particles known as Faraday circulation. By contrast, the merging of small heaps into larger ones—the coarsening process (Fig. 1)—is much less understood, and quantitative experiments have been scarce [5,6].

In the present study we introduce a model for the coarsening behavior, validated by experiments and detailed numerical simulations. This threefold approach leads to the identification of the average life span τ_N of the N -heap state as the proper coarsening quantity. It is proven to scale, in our 1D setup, as $\tau_N \propto N^{-3}$.

Experiments.—A glass box of dimensions $L \times H \times D = 300 \times 100 \times 2.1 \text{ mm}^3$ is vertically vibrated using a

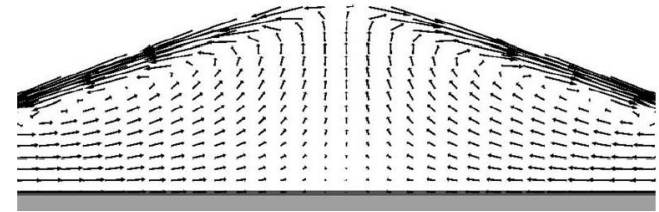
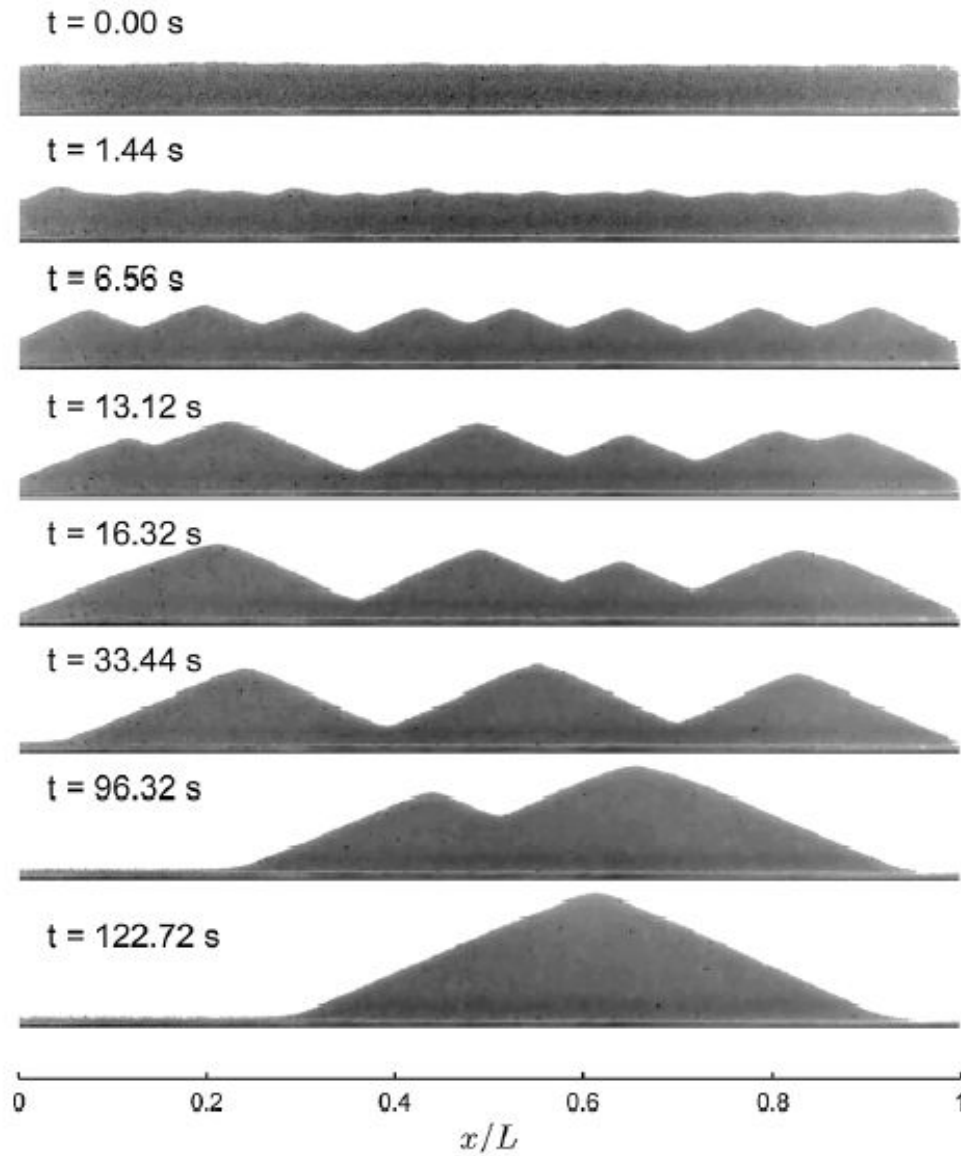


FIG. 2. Circulation in the Faraday heap: The arrows indicate the particle displacement (multiplied by 2 for clarity) during one driving cycle, averaged over 25 cycles. The part of the heap shown here corresponds to the black box in Fig. 1.

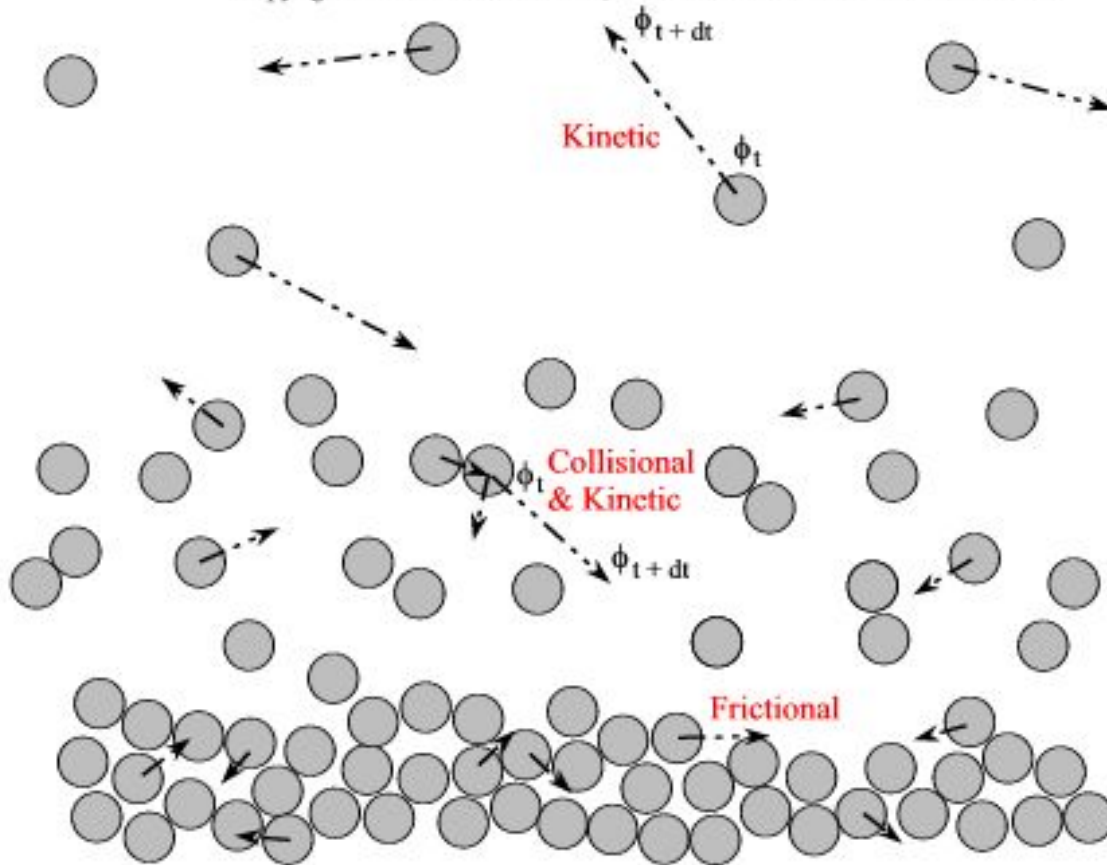
[van Gerner *et al.* 2007]

[van Gerner *et al.* 2009]

FIG. 1. Coarsening of a vertically vibrated 1D granular bed, as recorded in our experiments. It takes roughly two minutes to evolve from a flat landscape to a single Faraday heap. Every image is taken at the same point during the vibration cycle, when the container moves upward and the bed is pressed against the floor.

Interactions in granular medium

Copyright of Sebastien Dartevelle, from Sebastien Dartevelle's Ph.D. Thesis



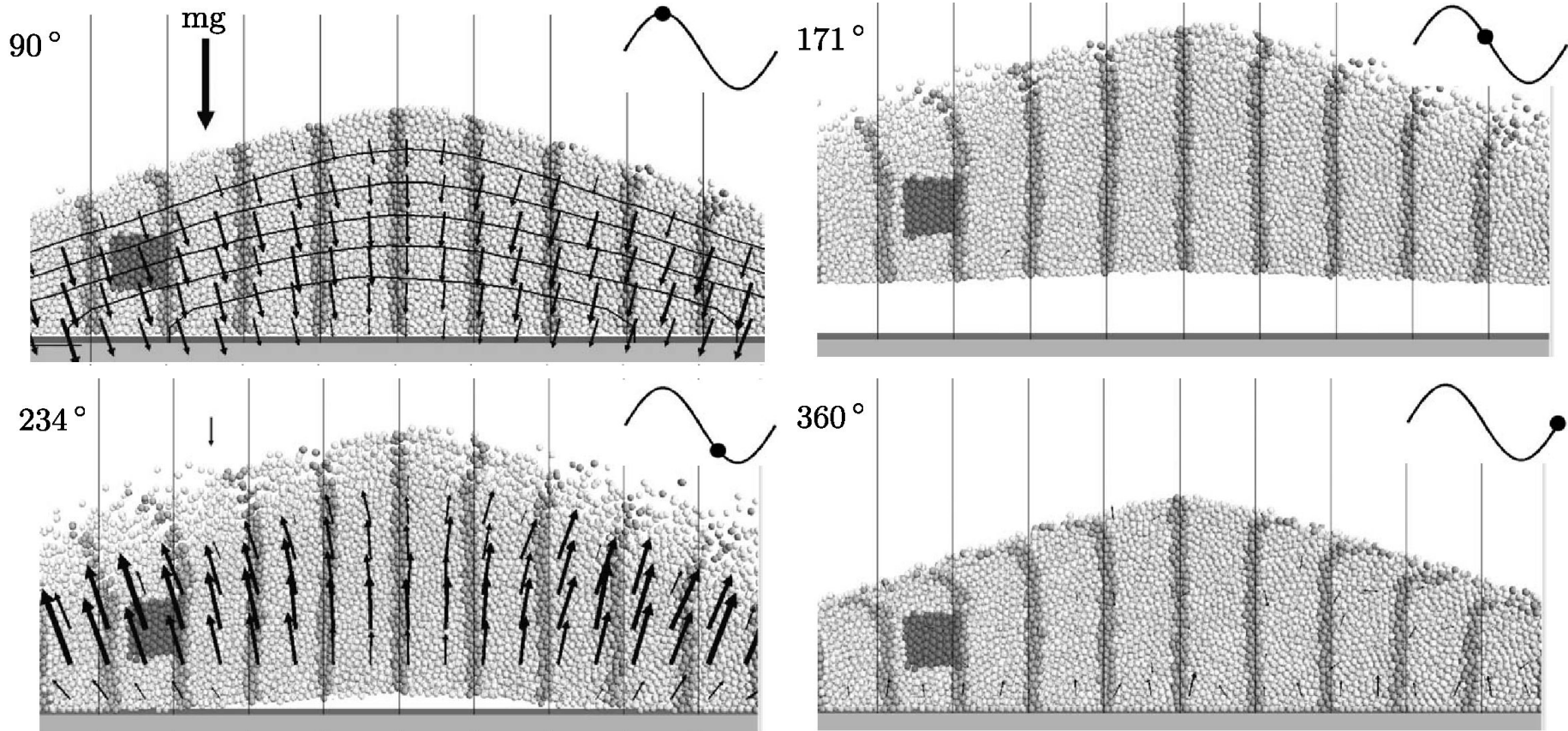
kinetic → dilute part of the flow, grains randomly translate; viscous dissipation and stress

collisional → higher concentration, grains also collide shortly; collisional dissipation and stress

frictional → very high concentration (>50% volume fraction), grains endure long, sliding and rubbing contacts

Earlier research...

[van Gerner *et al.* 2007]

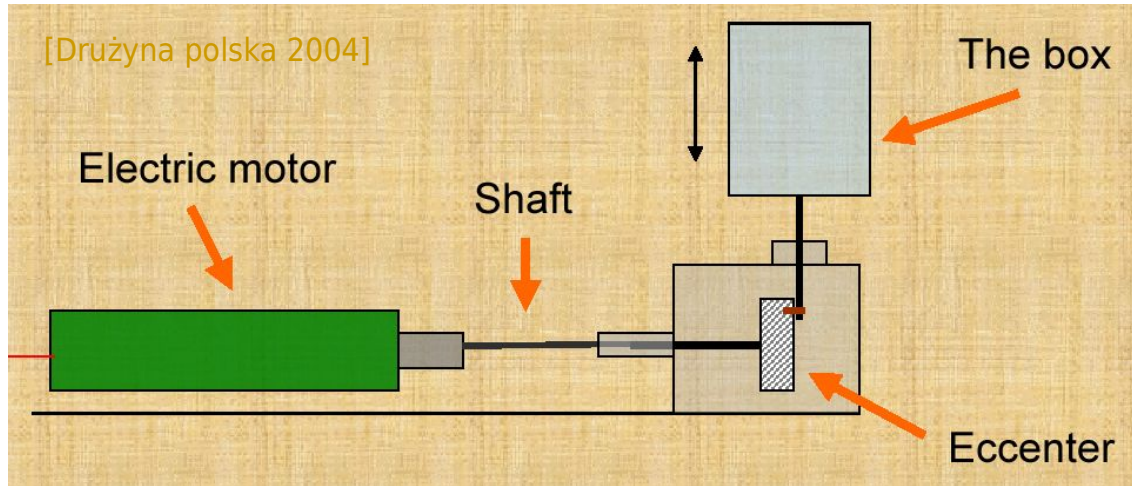


Stokesian forces (drag, air pressure) → drive particles **towards the center**
Newtonian forces (from collisions and gravity) → drive **particles outward**

IYPT history

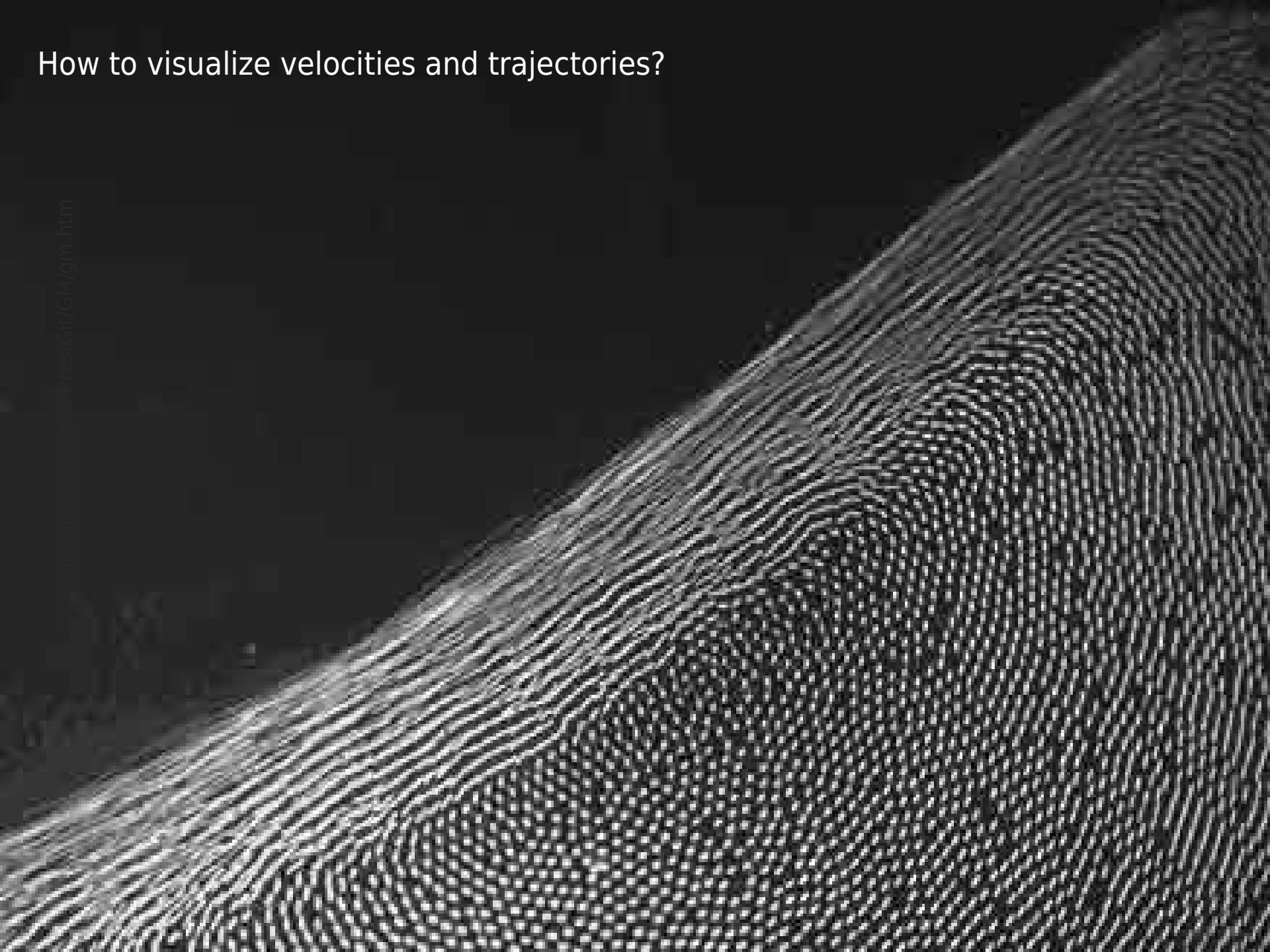
- **4. Self-formation of a pile (9th IYPT, 1996)**
 - A horizontal rigid plate vibrates vertically at a frequency of the order of 100 Hz. A cone-shaped pile of fine dispersed powder (e.g. Lycopodium or talc) which is heaped up on the plate remains stable at small amplitudes of the vibration. If the amplitude is increased the cone decays. Further increase of the amplitude yields a distribution confined by a sharp border and at still higher amplitudes a pile appears again. Investigate and explain this phenomenon.
- **16. Coloured sand (13th IYPT, 2000)**
 - Allow a mixture of differently coloured, granular materials to trickle into a transparent, narrow container. The materials build up in distinct bands. Investigate and explain this phenomenon.
- **7. Oscillating box (16th IYPT, 2003)**
 - Take a box and divide it into a number of small cells with low walls. Distribute some small steel balls between the cells. When the box is made to oscillate vertically, the balls occasionally jump from one cell to another. Depending on the frequency and the amplitude of the oscillation, the distribution of the balls can become stable or unstable. Study this effect and use a model to explain it.
- **15. Brazil Nut Effect (17th IYPT, 2004)**
 - When a granular mixture is shaken the larger particles may end up above the smaller ones. Investigate and explain this phenomenon. Under what conditions can the opposite distribution be obtained?

Engineering hints



- 1 - 10 Hz may be tricky to achieve with a loudspeaker
- No interest in Chladni figures? → vibration with a uniform amplitude
- Why not trying non-sinusoidal oscillations, never studied before systematically?

How to visualize velocities and trajectories?



http://ch3d.com/atom

More hints

- Tensor analysis :-/
 - but maybe it is possible to simplify everything with a rough, but clear theoretical approach? :-)
 - Numerical simulations can help a lot
 - What about making experiments and developing a theory for a 2D case? (grains are limited by two parallel glass plates)
 - Visualizing the motion and structure of heaps can be very helpful!
 - playing with exposure time for still images?
 - using colored grains?
 - making clear, informative videos (slow-motion?)
 - How far the heaps are reproducible, if everything is repeated? When does the system “forgets” initial conditions? Why?
 - How about reading more about relevant concepts from granular or soft matter physics, such as percolation, close packing, dynamic arrest?
-

Key questions

- Above all, what is the cause of the phenomenon?
 - Does the phenomenon appear with different granular materials (sand...)?
 - What parameters of the granular particles determine the shape of heaps? (**density?** **particle shape?** **average particle size?**) Do particles need to be monodisperse?
 - How relevant are the properties of air (density? viscosity?)
 - What are the relevant parameters during the shaking
 - amplitude, frequency, sine/non-sine oscillations?
 - Is there a way to describe the **3D shape** of heaps?
 - How exactly does the wave pattern **evolve in time**?
 - The heaps on initially almost plane layer. What is the **maximum possible amplitude** of the heaps?
 - How to model such a phenomenon experimentally? What are the basic requirements for the equipment?
 - Above all, what is your conclusion on the effect?
 - What new we can add to this profoundly researched problem?
-

Background reading (for courageous)

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- Igor S. Aranson, Lev S. Tsimring. Patterns and collective behavior in granular media: Theoretical concepts. [arXiv:cond-mat/0507419v1 \[cond-mat.soft\]](#)
- Vibrating sand table (from wysz, May 30, 2009, youtube), <http://www.youtube.com/watch?v=oDkQCfdRld8>
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- D. C. Rapaport. Subharmonic surface waves in vibrated granular media. Physica A 249, 232 (1998), <http://www.ph.biu.ac.il/~rapaport/papers/98b-phsa.pdf>

(a) what are its x , y , and z components in terms of r and θ ?

(b) Compute $(\hat{\mathbf{r}} \cdot \nabla)\hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ is the unit radial vector.

(c) For the functions in Prob. 1.15, evaluate $(\hat{\mathbf{r}} \cdot \nabla)f$.

Problem 1.22 (For masochists only.) Prove the definition of $(\mathbf{A} \cdot \nabla)\mathbf{B}$.

Problem 1.23 Derive the three quotient rules.

Problem 1.24

(a) Check that $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ and $\nabla \times (\nabla \phi) = 0$.

To work towards results?

- Nobody needs an infinitely perfect report in an infinite time!
 - If you cannot solve the entire problem, decide **what is really necessary** and solve a partial problem
 - If you can solve the entire problem, nevertheless **decide what partial case is sufficient, and your solution will be much better**
 - Be brave in what you do, but always reserve a great degree of scientific skepticism!
 - Procrastination is definitely a risk :-)
-

These problems have no solution?

- “But, my dear fellows,” said Feodor Simeonovich, having deciphered the handwriting. “This is Ben Beczalel's problem! Didn't Cagliostro prove that **it had no solution?**”
- “We know that it has no solution, too,” said Junta. “**But we wish to learn how to solve it.**”
- “How strangely you reason, Cristo... How can you look for a solution, where it does not exist? It's some sort of nonsense.”
- “Excuse me, Feodor, but it's you who are reasoning strangely. It's nonsense to look for a solution if it already exists. We are talking about how to deal with a problem that has no solution. This is a question of profound principle...”

Arkady Strugatsky and Boris Strugatsky

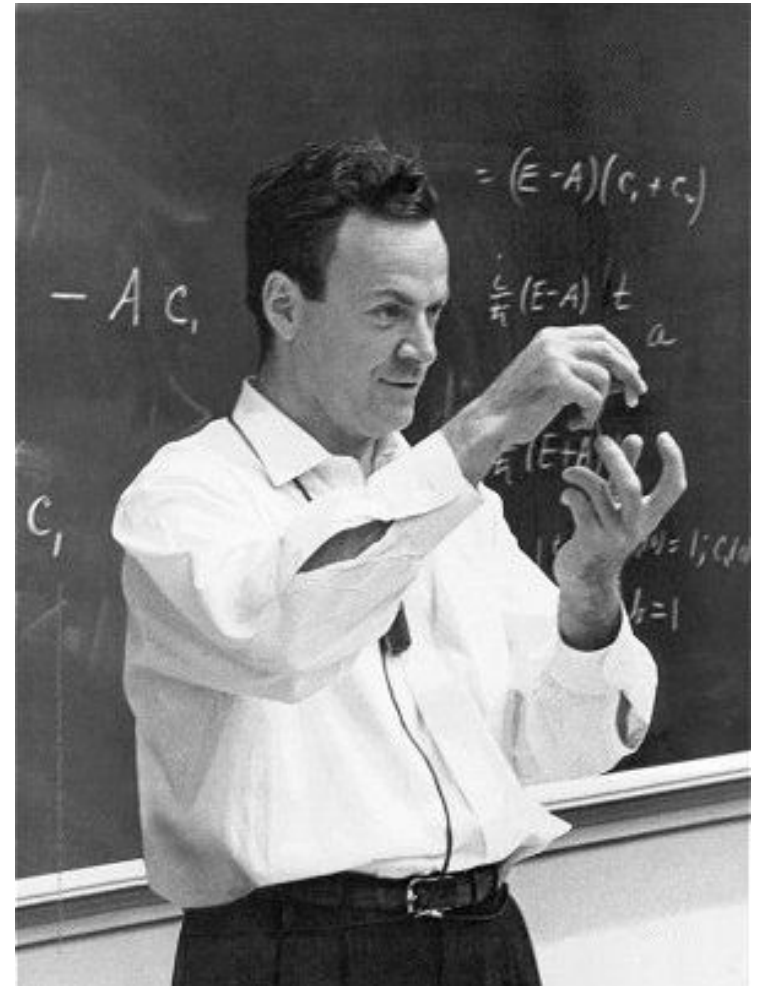
Quote from: Arkady Strugatsky and Boris Strugatsky. Monday Begins on Saturday. Translated from the Russian. (The Young Guard Publishing House, Moscow, 1966)

Habits and customs

- Originality and independence of your work is always considered as of a first priority
 - There is no “correct answer” to any of IYPT problems
 - Having a deep background knowledge about earlier work in a given field may certainly be a plus
 - Taking ideas without citing will seemingly be a serious misconduct
 - Critically distinguishing between personal contribution and common knowledge is likely to be appreciated
 - Reading more in a non-native language may be very helpful
 - Local libraries and institutions can always help in getting access to paid articles in journals, books and databases
 - Is IYPT all about reinventing the wheel, or innovating, creating, discovering, and being able to contrast own work with earlier knowledge and achievements of others?
 - Is IYPT all about competing, or about developing professional personal standards?
-

Feynman: to be self-confident?

- “I’ve very often made mistakes in my physics **by thinking the theory isn’t as good as it really is**, thinking that there are lots of complications that are going to spoil it
- — an attitude that anything can happen, in spite of what you’re pretty sure should happen.”





Turnaj mladých fyzikov 2011: poznámky k niekoľkým úlohám

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